MODELLING AND SIMULATION OF NATURAL CONVECTION HEAT EXCHANGER FOR DOMESTIC HEATING

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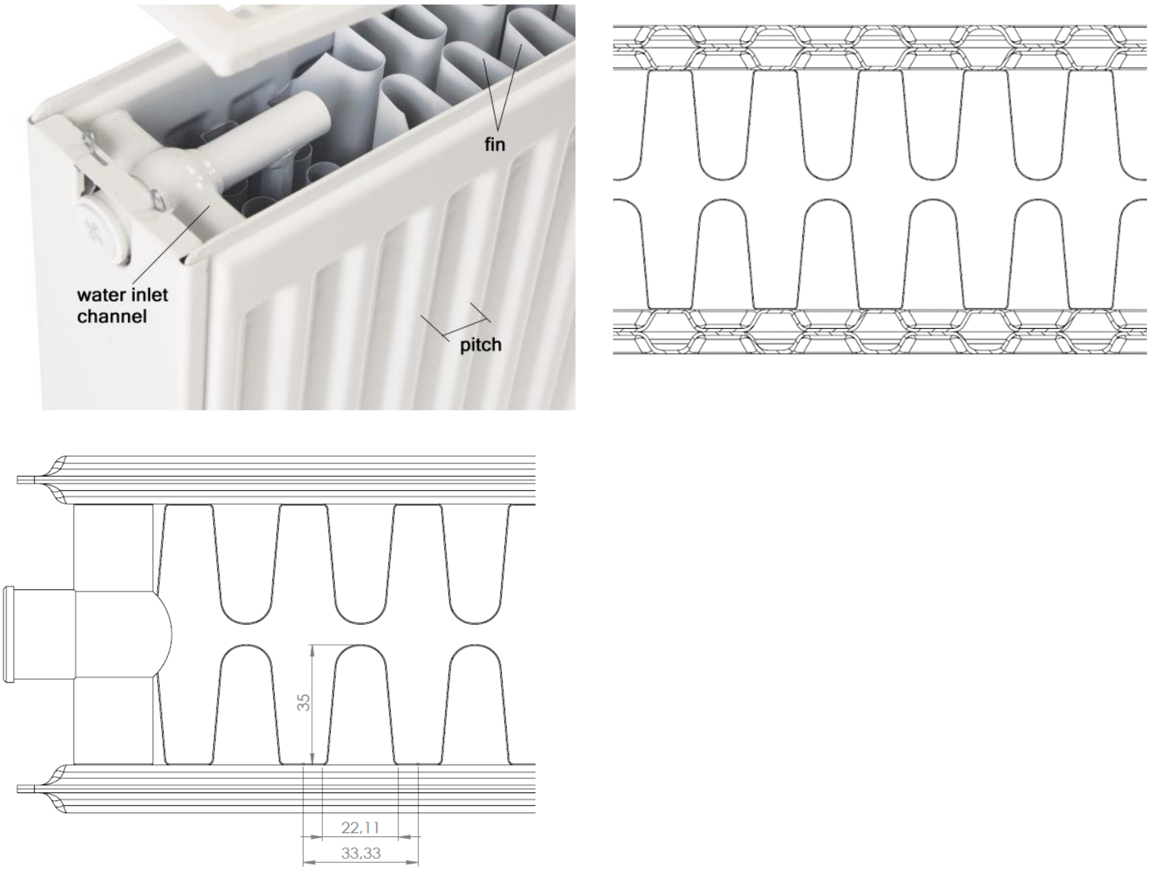
INTRODUCTION

Panel radiators are finned natural convective heat exchangers. General view of the device and fin profile is shown in the figures. Water flows inside chanels and outside air is heated up through finned and bare surfaces of heat exchanger. In order to simulate heat transfer mechanism in the radiator natural convection finite difference model is considered.



A finned heat exchanger: panel radiator

EQUATION OF STATES

In order to simulate the heat exchanger, equation of states and thermophysical properties such as viscosity, thermal convectivity is required. In radiator, watr flows inside channels, and air rise from outside through fins and panel through natuaral convective and radiative heating. For that purpaselet us investigate and model these properties as thefirst step tothe simulation process.

Air as a perfect gas

If an equation of state is given then all thermodynamic properties can be calculated by Consider an equation of state in the form of P(T,V)

$$ds = \left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv$$

where $v = \frac{V}{N}$ $s = \frac{S}{N}$ can be written.

If equation

$$C_v = C_v = T\left(\frac{\partial s}{\partial T}\right)_V$$

and Maxwel relation eqn

$$\left(\frac{\partial S}{\partial V}\right)_{T,N} = \left(\frac{\partial P}{\partial T}\right)_{V,N}$$

is used, equation becomes

$$ds = \frac{C_v(T)}{T} dT + \left(\frac{\partial P(T,v)}{\partial T}\right)_v dv$$

$$s = s_0 + \int_{T_0}^{T} \frac{C_v(T)}{T} dT + \int_{v_0}^{v} \left(\frac{\partial P(T,v)}{\partial T}\right)_v dv \quad (1.53a)$$

u equation of state 1.14 rewritten as $du = Tds - Pdv$ and above equation is substituted for ds

$$du = T\left(\frac{C_v}{T} dT + \left(\frac{\partial P(T,v)}{\partial T}\right)_v dv\right) - P(T,v)dv$$

$$du = C_v(T)dT + \left(T\left(\frac{\partial P(T,v)}{\partial T}\right)_v - P(T,v)\right) dv$$

integration of the equation gives

$$u = u_0 + \int_{T_0}^{T} C_v(T)dT + \int_{v_0}^{v} \left(T\left(\frac{\partial P(T,v)}{\partial T}\right)_v - P(T,v)\right) dv$$

Ideal gas equation of state:

$$P(T,V) = \frac{NRT}{V}$$

$$v = \frac{V}{N}$$

$$P(T,v) = \frac{RT}{v}$$

Where R=8.3145 kJ/(kmolK) is gas constant

$$s = s_0 + \int_{T_0}^{T} \frac{C_v(T)}{T} dT + \int_{v_0}^{v} \left(\frac{\partial P(T,v)}{\partial T}\right)_v dv$$

$$\left(\frac{\partial P(T,v)}{\partial T}\right) = \frac{R}{V}$$

$$s = s_0 + \int_{T_0}^{T} \frac{C_v(T)}{T} dT + \int_{v_0}^{v} \frac{R}{V} dv$$

$$s = s_0 + \int_{T_0}^{T} \frac{C_v(T)}{T} dT + Rln\frac{v}{v_0}$$

$$s = s_0 + \int_{T_0}^{T} \frac{C_p(T)}{T} dT - Rln\frac{P}{P_0}$$

$$u = u_0 + \int_{T_0}^{T} C_v(T)dT + \int_{v_0}^{v} \left(T \left(\frac{\partial P(T,v)}{\partial T} \right)_v - P(T,v) \right) dv$$

$$u = u_0 + \int_{T_0}^{T} C_v(T)dT + \int_{v_0}^{v} \left(T \left(\frac{R}{v} \right)_v - \frac{RT}{v} \right) dv$$

$$u = u_0 + \int_{T_0}^{T} C_v(T)dT$$

In order to create perfect gas thermodynamic properties of air, The first step is look Specific heat data for air the standart air mixture

Nitrogen	N_2	7.808400E+01	0.781121204
Oxygen	O_2	2.094600E+01	0.209535433
Argon	Ar	9.340000E-01	0.009343364
Carbondioxide	CO_2	3.970000E-02	
Neon	Ne	1.818000E-03	
Helium	He	5.240000E-04	
Methane	CH_4	1.790000E-04	
Water vapor	H_2O	0.00001-0.05	
		9.996400E+01	1

If it is assumed that Air is made of only N_2, O_2 and Ar, it will be changed to:

Name	Formula		% vol	M
Nitrogen	N2	78.084	0.781121204	28.014
Oxygen	O2	20.946	0.209535433	31.998
Argon	Ar	0.934	0.009343364	39.948
Air			1	28.96029

By finding specific heat data, we can able to establish the perfect gas equation of state. We will use Janaf tables from NIST (National Institute of Standards and Technology **Janaf.nist.gov)** to obtain specific heat data**.** After obtaining Specific heat data for Nitrogen, Oxygen and Argonne, Specific heat data for air is obtained by using ideal gas mixing rules. After the airspecific heat data is obtainedCubic spline interpolation will be applied to model air properties.

If a third degree polinomial is considered:

$$r_k(x) = a_k(x - x_k)^3 + b_k(x - x_k)^2 + c(x - x_k)^3 + y_k \quad 1 \le k \le n \quad (2.1.18)$$

In the interpolation proses polinoms should be passing through all data points

$$r_k(x_{k+1}) = y_{k+1} \quad 1 \le k \le n \quad (2.1.19)$$

In the same time the first derivative of the polynomial should also be continious while passing from one polynomial to the next one at the data point

$$r'_{k-1}(x_k) = r'_k(x_k) \quad 1 \le k \le n \quad (2.1.20)$$

For the third degree polinomial second derivative of the polynomial should also be continious while passing from one polynomial to the next one at the data point

$$r''_{k-1}(x_k) = r''_k(x_k) \quad 1 \le k \le n \quad (2.1.21)$$

All these conditions are not enough to solve the coefficients of the polinomials. Two more conditions are required. This two additional conditions (A and B of the following equation) can be given by user

$r''_1(x_1) = A \qquad r''_{n-1}(x_n) = B \quad (2.1.22)$

They are the second derivatives at the both hand of the series of polinomials. If A and B values are taken equals to 0, it is called a **natural cubic spline**. Other end conditions such as the ones depends one the first derivatives can also be set to solve the system of equations.
Defining $h_k = x_{k+1} - x_k \quad 1 \le k \le n \quad (2.1.23)$
System of equations become:

$$a_k h_k^3 + b_k h_k^2 + b_k h_k = y_{k+1} - y_k \;\; 1 \le k \le n \quad (2.1.24)$$

$$3a_{k-1} h_{k-1}^2 + 2b_{k-1} h_{k-1} + c_{k-1} - c_k = 0$$

$$6a_{k-1} h_{k-1} + 2b_{k-1} + 2b_k = 0$$

$$3b_0 = 0$$

$$6a_{n-1} h_{n-1} + 2b_{n-1} = 0$$

This set contains 3n-3 equations. This could a considerable load to the system of equation solving programs. To make calculation load simpler a special third degree polinomial can be considered. If our cubic polinomial is in the form of:

$$s_k(x) = a_k(x - x_k) + b_k(x_{k+1} - x) + [(x - x_k)^3 c_{k+1} + (x_{k+1} - x)^3 c_k]\,/(6h_k) \quad 1 \le k \le n \quad (2.1.25)$$

then derivative equations becomes

$$s'_k(x) = a_k - b_k + [(x - x_k)^2 c_{k+1} - (x_{k+1} - x)^2 c_k]\,/h_k \quad 1 \le k \le n \quad (2.1.26)$$

$$s"_k(x) = [(x - x_k) c_{k+1} - (x_{k+1} - x) c_k]\,/h_k \qquad 1 \le k \le n$$

a_k ve b_k coefficients can be expressed as a function of c_k

$$b_k = \frac{[6y_k - h_k c_k]}{6h_k} \quad 1 \le k \le n \quad (2.1.27)$$

$$a_k = \frac{[6y_{k+1} - h_k^2 c_{k+1}]}{6h_k} \quad 1 \le k \le n \quad (2.1.28)$$

In this case only c_k terms left in the system of equations to be solved.

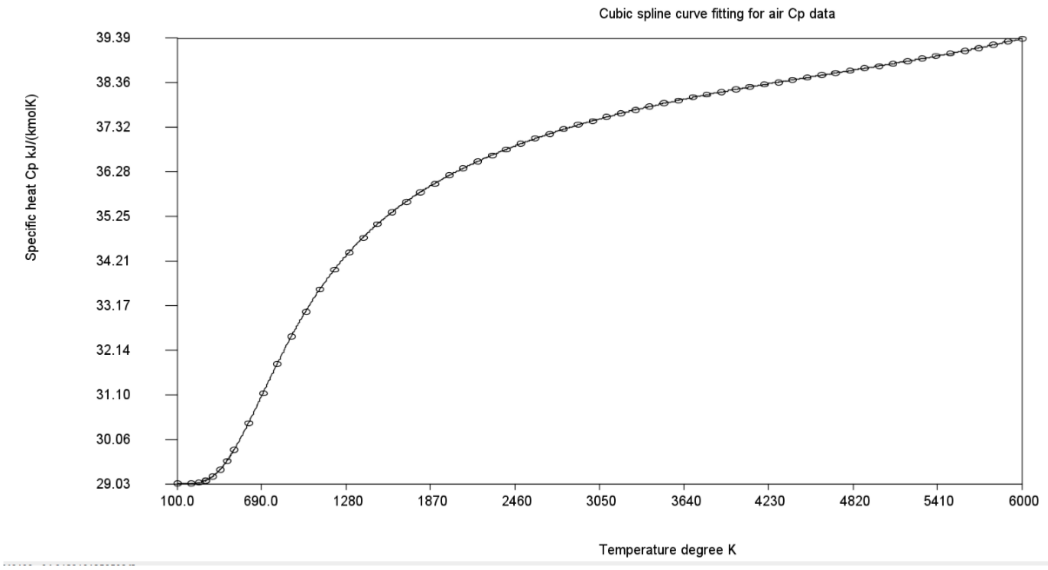
$$h_{k-1} c_{k-1} + 2(h_{k-1} - h_k) c_{k-1} + h_k c_{k+1} c_{k+1} = 6\left[\frac{y_{k+1} - y_k}{h_k} - \frac{y_k - y_{k-1}}{h_{k-1}}\right] \quad 1 \le k \le n \quad (2.1.29)$$

This system of equation has only n-2 terms to be solved. By making definition
$w_k = \frac{y_{k+1} - y_k}{h_k}, \quad 1 \le k \le n \;\; (2.1.31)$
System of equation becomes

$$\begin{bmatrix} 1 & & & & & & & \\ h_1 & 2(h_1 + h_2) & h_2 & & & & & \\ & h_2 & 2(h_2 + h_3) & & & & & \\ & \cdots \;\; \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\ & & & \cdots. & 2(h_{n-3} + h_{n-2}) & h_{n-2} & & \\ & & & & h_{n-2} & 2(h_{n-2} + h_{n-1}) & h_{n-1} & \\ & & & & & & 1 & \end{bmatrix} \begin{Bmatrix} c_0 \\ c_1 \\ c_2 \\ \cdots \\ c_{n-2} \\ c_{n-1} \\ c_n \end{Bmatrix} = \begin{Bmatrix} A \\ 6(w_2 - w_1) \\ 6(w_3 - w_2) \\ \cdots \\ 6(w_{n-2} - w_{n-3}) \\ 6(w_{n-1} - w_{n-2}) \\ B \end{Bmatrix} \quad (2.1.30)$$

Where A and B are the second derivative end conditions. A and B should be defined by user. Another important property of the above matrix is that it is a band matrix, therefore less amount of calculation is required to solve it (by using band matrix algorithms such as Thomas algorithm).

Cubic spline method has two advantages, the first is very accurate represantion of data, and the second one is ability to directly integrate and derivate the spline function.



Formulations of other thermophysical and thermodynamic properties

In order to calculate thermopysical properties (thermal conductivity and viscosity) of dry air Kadoya et al[135] equations are used. This equations has the following form:

$\eta_0(T_r) = A_0 T_r + A_1 T_r^{0.5} + A_2 + \frac{A_3}{T_r} + \frac{A_4}{T_r^2} + \frac{A_5}{T_r^3} + \frac{A_6}{T_r^4}$

$\Delta\eta(\rho_r) = \sum_{i=1}^{4} B_i \rho_r^i$

$\eta(T_r, \rho_r) = H[\eta_0(T_r) + \Delta\eta(\rho_r)]$

$k_0(T_r) = C_0 T_r + C_1 T_r^{0.5} + C_2 + \frac{C_3}{T_r} + \frac{C_4}{T_r^2} + \frac{C_5}{T_r^3} + \frac{C}{T_r^4}$

$\Delta k(\rho_r) = \sum_{i=1}^{4} D_i \rho_r^i$

$k(T_r, \rho_r) = \Lambda[k_0(T_r) + \Delta k(\rho_r)]$

Where $\rho_r = \rho/\rho^*$ $\quad T_r = T/T^*$

Coefficients of the equations are given in Table

Table Coefficients of viscosity and thermal conductivity equations

$T^* = 132.5$ K	$\rho^* = 314.3$ kg/m3	$\Lambda = 25.9778\ (10^{-3} W/(mK)$	H=6.1609 (10^{-6} Pas)	
i	A_i	B_i	C_i	D_i
0	0.128517	0.465601	0.239503	0.402287
1	2.60661	1.26469	0.00649768	0.356603
2	-1	-0.511425	1	-0.163159
3	-0.709661	0.2746	-1.92615	0.138059
4	0.662534		2.00383	-0.0201725
5	-0.197846		-1.07553	
6	0.00770147		0.229414	

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$k_0(T_r) = C_0 T_r + C_1 T_r^{0.5} + C_2 + \frac{C_3}{T_r} + \frac{C_4}{T_r^2} + \frac{C_5}{T_r^3} + \frac{C}{T_r^4}$

$\Delta k(\rho_r) = \sum_{i=1}^{4} D_i \rho_r^i$

$k(T_r, \rho_r) = \Lambda[k_0(T_r) + \Delta k(\rho_r)]$

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Properties of steam (water)

In recent years maximum operating temperatures and pressures of Rankine cycle power plants has increased. International Association for the Properties of Water and Steam(IAPWS) is developed a new set of equation of states which are more accurate and covers larger range of data. This new set of equations are developed in 1997[59]. Steam properties are given by 5 sets of equation of states, as shown in the Figure

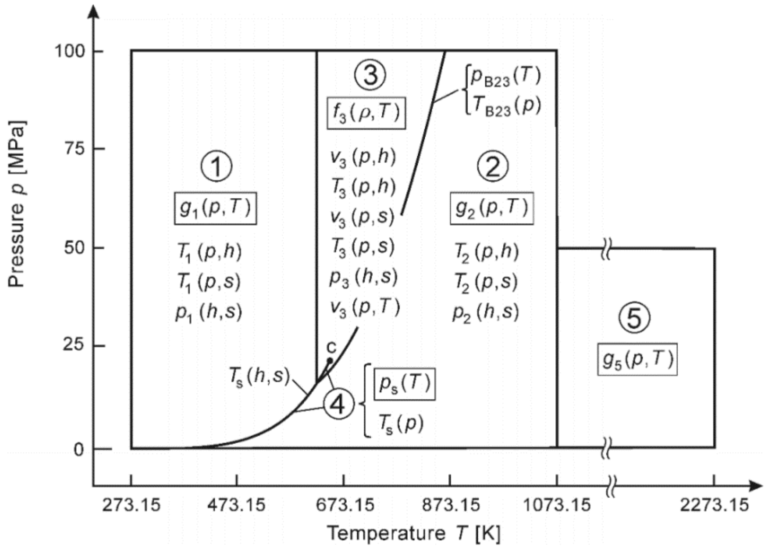


Figure IAPWS 97 Equation of state regions for steam

The first equation, which covers basically liquid region has the following gibbs free energy form:

$$\frac{g_1(P,T)}{RT} = \gamma(\pi,\tau) = \sum_{i=1}^{34} n_i (71-\pi)^{I_i} (\tau - 1222)^{J_i}$$

Where $\pi = \frac{P}{P^*}$ $\quad \tau = \frac{T^*}{T}$ $\quad p^*$=16.62 MPa and T^*=1386 K R=0461526 kJ/(kgK)

Table 2.6.4 coefficients of eqn. 2.6.11

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	0	-2	0.14632971213167	18	2	3	-4.4141845331E-06
2	0	-1	-0.84548187169114	19	2	17	-7.2694996298E-16

3	0	0	-3.75636036720400	20	3	-4	-3.1679644845E-05
4	0	1	3.38551691683850	21	3	0	-2.8270797985E-06
5	0	2	-0.95791963387872	22	3	6	-8.5205128120E-10
6	0	3	0.15772038513228	23	4	-5	-2.2425281908E-06
7	0	4	-0.01661641719950	24	4	-2	-6.5171222896E-07
8	0	5	0.00081214629984	25	4	10	-1.4341729938E-13
9	1	-9	0.00028319080124	26	5	-8	-4.0516996860E-07
10	1	-7	-0.00060706301566	27	8	-11	-1.2734301742E-09
11	1	-1	-0.01899006821842	28	8	-6	-1.7424871231E-10
12	1	0	-0.03252974877051	29	21	-29	-6.8762131296E-19
13	1	1	-0.02184171717541	30	23	-31	1.4478307829E-20
14	1	3	-0.00005283835797	31	29	-38	2.6335781663E-23
15	2	-3	-0.00047184321073	32	30	-39	-1.1947622640E-23
16	2	0	-0.00030001780793	33	31	-40	1.8228094581E-24
17	2	1	0.00004766139391	34	32	-41	-9.3537087292E-26

Thermodynamic relations can be calculated from these thermodynamic relations
Specific volume: $v = \left(\frac{\partial g}{\partial P}\right)_T$ (2.6.12)
Specific enthalpy: $h = g - T\left(\frac{\partial g}{\partial T}\right)_P$ (2.6.13)
Specific internal energy: $u = g - T\left(\frac{\partial g}{\partial T}\right)_P - P\left(\frac{\partial g}{\partial P}\right)_T$ (2.6.14)
Specific entropy: $s = \left(\frac{\partial g}{\partial T}\right)_P$ (2.6.15)
Specific isobaric heat capacity: $C_p = \left(\frac{\partial h}{\partial T}\right)_P$ (2.6.16)
Specific isochoric heat capacity: $C_v = \left(\frac{\partial u}{\partial T}\right)_v$ (2.6.17)

The second equation equation, which covers vapor region has the following gibbs free energy form:

$\frac{g_2(P,T)}{RT} = \gamma(\pi,\tau) = \gamma^0(\pi,\tau) + \gamma^r(\pi,\tau)$ (2.6.18)

Where $\pi = \frac{P}{P^*}$ $\tau = \frac{T^*}{T}$ R=0.461526 kJ/(kgK), $\gamma^0(\pi,\tau)$is the ideal gas part of EOS, and $\gamma^r(\pi,\tau)$is the real gas departure the EOS. İdeal gas part equation:

$$\gamma^0(\pi,\tau) = \ln(\pi) + \sum_{i=1}^{9} n_i^0 \tau^{J_i} \quad (2.6.19)$$

Where P^*=1MPa and T^*=540 K
Table 2.6.5 coefficients of eqn. 2.6.19

i	J_i	n_i^0	i	J_i	n_i^0
1	0	-9.692768650E+00	6	-2	1.4240819171E+00
2	1	1.008665597E+01	7	-1	-4.3839511319E+00
3	-5	-5.608791128E-03	8	2	-2.8408632461E-01
4	-4	7.145273808E-02	9	3	2.1268463753E-02
5	-3	-4.071049822E-01			

dimensionless residual part of the basic equation $g2(p,T)$ is as follows:

$$\gamma^r(\pi,\tau) = \sum_{i=1}^{43} n_i \pi^{I_i} (\tau - 0.5)^{J_i} \quad (2.6.20)$$

Where P^*=1MPa and T^*=540 K
Table 2.6.6 coefficients of eqn. 2.6.20

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	1	0			7	0	-5.9059564324270E-18
2	1	1	-1.7834862292358E-02	24	7	11	-1.2621808899101E-06
3	1	2	-4.5996013696365E-02	25	7	25	-3.8946842435739E-02
4	1	3	-5.7581259083432E-02	26	8	8	1.1256211360459E-11
5	1	6	-5.0325278727930E-02	27	8	36	-8.2311340897998E+00
6	2	1	-3.3032641670203E-05	28	9	13	1.9809712802088E-08
7	2	2	-1.8948987516315E-04	29	10	4	1.0406965210174E-19
8	2	4	-3.9392777243355E-03	30	10	10	-1.0234747095929E-13
9	2	7	-4.3797295650573E-02	31	10	14	-1.0018179379511E-09

10	2	36	-2.6674547914087E-05	32	16	29	-8.0882908646985E-11
11	3	0	2.0481737692309E-08	33	16	50	1.0693031879409E-01
12	3	1	4.3870667284435E-07	34	18	57	-3.3662250574171E-01
13	3	3	-3.2277677238570E-05	35	20	20	8.9185845355421E-25
14	3	6	-1.5033924542148E-03	36	20	35	3.0629316876232E-13
15	3	35	-4.0668253562649E-02	37	20	48	-4.2002467698208E-06
16	4	1	-7.8847309559367E-10	38	21	21	-5.9056029685639E-26
17	4	2	1.2790717852285E-08	39	22	53	3.7826947613457E-06
18	4	3	4.8225372718507E-07	40	23	39	-1.2768608934681E-15
19	5	7	2.2922076337661E-06	41	24	26	7.3087610595061E-29
20	6	3	-1.6714766451061E-11	42	24	40	5.5414715350778E-17
21	6	16	-2.1171472321355E-03	43	24	58	-9.4369707241210E-07
22	6	35	-2.3895741934104E+01				

Region 3 equation is given as Helmholts free energy form:

$$\frac{f_3(\rho,T)}{RT} = \phi(\delta,\tau) = n_1 \ln(\delta) + \sum_{i=2}^{40} n_i \delta^{I_i} \tau^{J_i} \quad (2.6.21)$$

Where $\delta = \frac{\rho}{\rho^*}$ $\tau = \frac{T^*}{T}$, $T^*=T_c=647.096$ and R=0461526 kJ/(kgK)

Table 2.6.4 coefficients of eqn. 2.6.11

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	0	0	1.065807002851E+00	21	3	4	-2.0189915023570E+00
2	0	0	-1.573284529024E+01	22	3	16	-8.2147637173963E-03
3	0	1	2.094439697431E+01	23	3	26	-4.7596035734923E-01
4	0	2	-7.686770787872E+00	24	4	0	4.3984074473500E-02
5	0	7	2.618594778795E+00	25	4	2	-4.4476435428739E-01
6	0	10	-2.808078114862E+00	26	4	4	9.0572070719733E-01
7	0	12	1.205336969652E+00	27	4	26	7.0522450087967E-01
8	0	23	-8.456681281250E-03	28	5	1	1.0770512626332E-01
9	1	2	-1.265431547771E+00	29	5	3	-3.2913623258954E-01
10	1	6	-1.152440780668E+00	30	5	26	-5.0871062041158E-01
11	1	15	8.852104398432E-01	31	6	0	-2.2175400873096E-02
12	1	17	-6.420776518161E-01	32	6	2	9.4260751665092E-02
13	2	0	3.849346018667E-01	33	6	26	1.6436278447961E-01
14	2	2	-8.521470882421E-01	34	7	2	-1.3503372241348E-02
15	2	6	4.897228154188E+00	35	8	26	-1.4834345352472E-02
16	2	7	-3.050261725697E+00	36	9	2	5.7922953628084E-04
17	2	22	3.942053687915E-02	37	9	26	3.2308904703711E-03
18	2	26	1.255840842431E-01	38	10	0	8.0964802996215E-05
19	3	0	-2.799932969871E-01	39	10	1	-1.6557679795037E-04
20	3	2	1.389979956946E+00	40	11	26	-4.4923899061815E-05

It should be noted that this set of equation is function of density and temperature, and basic equation is helmholts equation so, let us list definition of other thermodynamic properties

Pressure: $P = \rho^2 \left(\frac{\partial f}{\partial \rho}\right)_T$

Specific enthalpy: $h = f - T\left(\frac{\partial f}{\partial T}\right)_p + \rho\left(\frac{\partial f}{\partial \rho}\right)_T$

Specific internal energy: $u = f - T\left(\frac{\partial f}{\partial T}\right)_p$

Specific entropy: $s = \left(\frac{\partial f}{\partial T}\right)_\rho$

Specific isobaric heat capacity: $C_p = \left(\frac{\partial h}{\partial T}\right)_p$

Specific isochoric heat capacity: $C_v = \left(\frac{\partial u}{\partial T}\right)_v$

Region 4 of the equation defines saturation region. The basic equation is given as a polynomial

$$\beta^2\vartheta^2 + n_1\beta^2\vartheta + n_2\beta^2 + n_3\beta\vartheta^2 + n_4\beta\vartheta + n_5\beta + n_6\vartheta^2 + n_7\vartheta + n_8 = 0 \quad (2.6.18)$$

Where

$\beta = \left(\frac{P_S}{P^*}\right)^{0.25}$

$\vartheta = \frac{T_S}{T^*} + \frac{n_9}{\left(\frac{T_S}{T^*}\right) - n_{10}}$

From this equation both saturation pressure and saturation temperature equation can be derived.

$\frac{P_S}{P^*} = \left[\frac{2C}{-B + (B^2 - 4AC)^{0.5}}\right]^4$

Where $P^*=1$ MPa

$A = \vartheta^2 + n_1\vartheta + n_2$
$B = n_3\vartheta^2 + n_4\vartheta + n_5$
$C = n_6\vartheta^2 + n_7\vartheta + n_8$

Table coefficients of eqn

i	n_i	i	n_i
1	1.1670521453E+03	6	1.4915108614E+01
2	-7.2421316703E+05	7	-4.8232657362E+03
3	-1.7073846940E+01	8	4.0511340542E+05
4	1.2020824702E+04	9	-2.3855557568E-01
5	-3.2325550322E+06	10	6.5017534845E+02

It is also possible to drive saturation temperature equation from the basic polynomial as:

$$\frac{T_s}{T^*} = \frac{n_{10} + D - [(n_{10} + D)^2 - 4(n_9 + n_{10}D)]^{0.5}}{2} \quad (2.6.22)$$

Where T^*=1 K

$$D = \frac{2G}{-F - (F^2 - 4EG)^{0.5}}$$
$$E = \beta^2 + n_3\beta + n_6$$
$$F = n_1\beta^2 + n_4\beta + n_7$$
$$G = n_2\beta^2 + n_5\beta + n_8$$

And the final region for steam is region 5, again given as gibbs free equation type EOS

$\frac{g_5(P,T)}{RT} = \gamma(\pi,\tau) = \gamma^0(\pi,\tau) + \gamma^r(\pi,\tau)$ (2.6.23)

Where $\pi = \frac{P}{P^*}$ $\tau = \frac{T^*}{T}$ R=0.461526 kJ/(kgK), $\gamma^0(\pi,\tau)$is the ideal gas part of EOS, and $\gamma^r(\pi,\tau)$is the real gas difference of the EOS. İdeal gas part equation:

$\gamma^0(\pi,\tau) = \ln(\pi) + \sum_{i=1}^{9} n_i^0\tau^{J_i}$ (2.6.24)

Where p^*=1MPa and T^*=1000 K

Table coefficients of eqn.

i	J_i^0	n_i^0	i	J_i^0	n_i^0
1	0	-13.1799836742	4	-2	0.3690153498
2	1	6.8540841634	5	-1	-3.1161318214
3	-3	-0.0248051489	6	2	-0.3296162654

The real gas part of the equation

$$\gamma^r(\pi,\tau) = \sum_{i=1}^{43} n_i\pi^{I_i}\tau^{J_i} \quad (2.6.25)$$

Table coefficients of eqn. 2.6.25

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	1	1	1.5736404855E-03	4	2	3	2.2440037409E-06
2	1	2	9.0153761674E-04	5	2	9	-4.1163275453E-06
3	1	3	-5.0270077678E-03	6	3	7	3.7919454823E-08

Formulations of other thermophysical and thermodynamic properties

In order to calculate thermopysical properties (thermal conductivity and viscosity) of dry air Kadoya et al[135] equations are used. This equations has the following form:

$\eta_0(T_r) = A_0T_r + A_1T_r^{0.5} + A_2 + \frac{A_3}{T_r} + \frac{A_4}{T_r^2} + \frac{A_5}{T_r^3} + \frac{A_6}{T_r^4}$

$\Delta\eta(\rho_r) = \sum_{i=1}^{4} B_i\rho_r^i$

$\eta(T_r,\rho_r) = H[\eta_0(T_r) + \Delta\eta(\rho_r)]$

$k_0(T_r) = C_0T_r + C_1T_r^{0.5} + C_2 + \frac{C_3}{T_r} + \frac{C_4}{T_r^2} + \frac{C_5}{T_r^3} + \frac{C}{T_r^4}$

$\Delta \mathrm{k}(\rho_r) = \sum_{i=1}^{4} D_i \rho_r^i$

$\mathrm{k}(T_r, \rho_r) = \Lambda[k_0(T_r) + \Delta\mathrm{k}(\rho_r)]$

Where $\rho_r = \rho/\rho^*$ $\quad T_r = T/T^*$

Coefficients of the equations are given in Table

Table Coefficients of viscosity and thermal conductivity equations

$T^* = 132.5$ K	$\rho^* = 314.3$ kg/m3	$\Lambda = 25.9778\ (10^{-3} W/(mK)$	H=6.1609 (10^{-6} Pas)	
i	A_i	B_i	C_i	D_i
0	0.128517	0.465601	0.239503	0.402287
1	2.60661	1.26469	0.00649768	0.356603
2	-1	-0.511425	1	-0.163159
3	-0.709661	0.2746	-1.92615	0.138059
4	0.662534		2.00383	-0.0201725
5	-0.197846		-1.07553	
6	0.00770147		0.229414	

Viscosity and thermal conductivity values of steam and water are taken from IAPWS Industrial Formulation 1997[15]. This equations are as follows:

Viscosity equations:

$\eta(\rho,T)=\psi(\delta,\theta) = \eta^*[\psi_0(\theta)\psi_1(\delta,\theta)]$

Where $\eta^* = 10^{-6}\ Pas \quad \delta = \frac{\rho}{\rho^*} \quad \theta = T/T^*$

with $T^* = T_c = 647.096\ K \ \ \rho^* = \rho_c = 322\ kg/m^3$

$\psi_0(\theta) = \theta^{0.5}\left[\sum_{i=1}^{4} n_i^0\, \theta^{1-i}\right]^{-1}$ Coefficients of equation given below:

Table 3.2 Coefficients of equation

i	n_i^0
1	0.167752e-1
2	0.220462e-1
3	0.6366564e-2
4	-0.241605e-2

$\psi_1(\delta,\theta) = exp\left[\delta \sum_{i=1}^{21} n_i\,(\delta-1)^{I_i}\left(\frac{1}{\theta}-1\right)^{J_i}\right]$

Table Coefficients of equation

i	**I_i**	**J_i**	**N_i**	**i**	**I_i**	**J_i**	**N_i**
1	0	0	5.200940E-01	12	2	2	-7.724790E-01
2	0	1	8.508950E-02	13	2	3	-4.898370E-01
3	0	2	-1.083740E+00	14	2	4	-2.570400E-01
4	0	3	-2.895550E-01	15	3	0	1.619130E-01
5	1	0	2.225310E-01	16	3	1	2.573990E-01
6	1	1	9.991150E-01	17	4	0	-3.253720E-02
7	1	2	1.887970E+00	18	4	3	6.984520E-02
8	1	3	1.266130E+00	19	5	4	8.721020E-03
9	1	5	1.205730E-01	20	6	3	-4.356730E-03
10	2	0	-2.813780E-01	21	6	5	-5.932640E-04
11	2	1	-9.068510E-01				

Thermal conductivity equations

$$\frac{\mathrm{k}(\rho,T)}{\lambda^*} = \Lambda(\delta,\theta)=\Lambda_0(\theta)+\Lambda_1(\delta)+\Lambda_2(\delta,\theta)$$

$$\Lambda_0(\theta)=\theta^{0.5}\sum_{i=1}^{4} n_i^0\,\theta^{i-1}$$

Table Coefficients of equation

i	n_i^0
1	0.102811e-1
2	0.299621e-1
3	0.156146e-1

4	-0.422464e-2

$\Lambda_1(\delta)=n_1+n_2\delta+n_3 exp[n_4(\delta+n_5)^2]$

Table Coefficients of equation

i	n_i
1	0.39707
2	0.400302
3	-0.171587e4
4	-0.239219e1

$\Lambda_2(\delta,\theta)=(n_1\theta^{-10}+n_2)\delta^{1.8}exp[n_2(1-\delta^{2.8})]+n_4A\delta^B exp\left[\left(\frac{B}{1+B}\right)(1-\delta^{1+B})\right]+$
$n_5 exp[n_6\theta^{1.5}+n_7\delta^{-5}]$

$$A(\theta)=2+n_8(\Delta\theta)^{-0.6}$$

$B(\theta)=\begin{cases}(\Delta\theta)^{-1} & for\ \theta\geq 1\\ n_9(\Delta\theta)^{-0.6} & for\ \theta<1\end{cases}$ 3.27b with $\Delta\theta=|\theta-1|+n_{10}$

Table Coefficients of equation

i	**n_i**	**i**	**n_i**
1	7.013090E-02	6	-4.117170E+00
2	1.185200E-02	7	-6.179370E+00
3	6.428570E-01	8	8.229940E-02
4	1.699370E-03	9	1.009320E+01
5	-1.020000E+00	10	3.089760E-03

HEAT TRANSFER EQUATIONS

Internal flow :

Water is flowing inside channels. Water cross sectional area is not circular, therefore hydrolic diameter concept is used.

Hydrolic diameter

$$D_H=\frac{4A}{P}$$

One phase pressure drop

Goudar- Sonnad equation (2008) Valid region: all values

$a=\frac{2}{\ln(10)}$

$b=\frac{(\varepsilon/D)}{3.7}$

$d=\frac{ln(10)}{5.02}Re$

$s=bd+\ln\left(\frac{d}{q}\right);$

$q=s^{\left(\frac{s}{s+1}\right)}$

$g=bd+\ln\left(\frac{d}{q}\right)$

$z=\frac{q}{g}$

$\delta_{LA}=\frac{g}{g+1}z$

$\delta_{CFA}=\delta_{LA}\left(1+\frac{z/2}{(g+1)^2+(\frac{z}{3})(2g-1)}\right)$

$\frac{1}{\sqrt{f}} = a\left[ln\left(\frac{d}{q}\right) + \delta_{CFA}\right]$

Laminar flow heat transfer:

$Nu = 3.66$

Heat transfer equations **Fully developed transitional/intermittent region** T_s=const

Abraham-Sparrow-Tong[34] equation

$Nu = 2.2407\left(\frac{Re}{1000}\right)^4 - 29.499\left(\frac{Re}{1000}\right)^3 + 142.32\left(\frac{Re}{1000}\right)^2 - 292.51\left(\frac{Re}{1000}\right) + 219.88 \quad 2300 \leq Re \leq 3100$

Abraham recommended Gnilenski equation to be used above Re>3100

Gnielinski[33] equation

$Nu = \frac{\left(\frac{f}{8}\right)(Re-1000)Pr}{1.07+12.7\left(\frac{f}{8}\right)^{.5}\left(Pr^{\frac{2}{3}}-1\right)} \quad 0.5 \leq Pr \leq 2000 \quad 2300 \leq Re \leq 510^6$

External natural convection:

Churchill & Chu Equation for all Ra range[36] (valid both turbulent and laminar cases)

Rayleigh Number: $\mathrm{Ra}_x = \mathrm{Gr}_x Pr = \frac{g\beta(T_s-T_\infty)x^3}{\nu\alpha} \quad \alpha = \frac{k}{\rho C_p}$

Critical Rayleigh Number $\mathrm{Ra}_{x,critical} = 10^9$

$\mathrm{Nu}_L = 4/3\mathrm{Nu}_x$

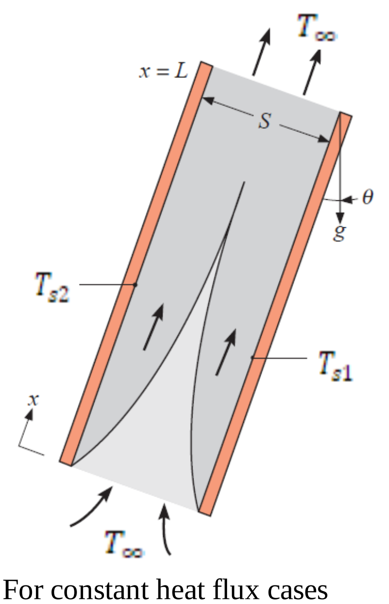
$\mathrm{Nu}_L = \left\{0.825 + \frac{0.387Ra_L^{1/6}}{\left[1+\left(\frac{0.492}{\mathrm{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2$

Natural convection heat transfer in channels

For symmetrical heated, isothermal plates Elenbaas[58] equation is as folows:

$\mathrm{Nu}_S = \left(\frac{q/A}{T_s-T_\infty}\right)\frac{S}{k} = \frac{1}{24}Ra_S\left(\frac{S}{L}\right)\left[1 - exp\left(-\frac{35}{Ra_S\left(\frac{S}{L}\right)}\right)\right]^{3/4}$

where **Rayleigh Number:** $\mathrm{Ra}_S = \frac{g\beta(T_s-T_\infty)S^3}{\nu\alpha} \quad 10^{-1} \leq \mathrm{Ra}_S \leq 10^5$



For constant heat flux cases

$\mathrm{Nu}_S = \left(\frac{q"_s}{T_s-T_\infty}\right)\frac{S}{k} \quad \mathrm{Ra}_S^* = \frac{g\beta q"_s S^4}{k\nu\alpha}$

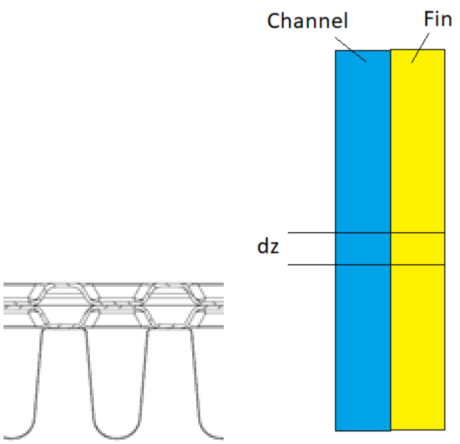
for symmetric fully developed constant heat flux
$\mathrm{Nu}_{SL} = 0.144[\mathrm{Ra}_S^*(S/L)]^{1/2}$
for asymmetric fully developed constant heat flux
$\mathrm{Nu}_{SL} = 0.204[\mathrm{Ra}_S^*(S/L)]^{1/2}$
Bar-Cohen-Rohsenow[59] equation:
For isothermal plates

$$\mathrm{Nu}_{SL} = \left[\frac{576}{\left(Ra_S\left(\frac{S}{L}\right)^2\right)} + \frac{2.87}{\left(Ra_S\left(\frac{S}{L}\right)^{1/2}\right)}\right]^{-1/2} \quad 10 \le \mathrm{Ra}_S \le 100 \quad T_{s1} = T_{s2} \text{ symmetric isothermal}$$

$$\mathrm{Nu}_{SL} = \left[\frac{144}{\left(Ra_S\left(\frac{S}{L}\right)^2\right)} + \frac{2.87}{\left(Ra_S\left(\frac{S}{L}\right)^{1/2}\right)}\right]^{-1/2} \quad 10 \le \mathrm{Ra}_S \le 100 \quad T_{s1},\ q_{s2}" = 0 \text{ isothermal adiabatic}$$

SIMULATION MODELLING

We will consider a single fin and a single channel section. In this case fin will be exposed from inside region to a channel natural convection and from the output section a vertical wall natural convection which values differs. The temperature profiles of internal and external parts will also be differs.



In vertical site Finite difference heat transfer equations starts from buttom, where air inlet temperature is the room temperature. Water inlet temperature and water exit temperature are given. Mass flow rateof water flow in channels are unknown. We will try to find the required flow rates correspondes to water inlet-outlet conditions. Finite difference heat transfer equations:

Finite difference length:

$$dz = \frac{L}{N}$$

Where L= radiator height

N= number of finite difference division

$$dQ_i = U_i dA_i (T_{water} - T_{air})$$

$$\frac{1}{U_i} = \frac{1}{h_{water\,i}} + \frac{t_{panel}}{k_{panel}} + \frac{1}{\eta_{fin\,i} h_{air\,i}}$$

$$M = \frac{2hl_{fin}}{kt_{fin}}$$

$$\eta_{fin} = \left(l_{fin}\frac{\tanh{(M)}}{m} + lbase\right)\frac{1}{(l_{fin} + l_{base})}$$

$$dQ_i = m_{water}C_{p_{water}}(T_{water\ i})(T_{water\ i} - T_{water\ i+1})$$

$$dQ_i = dQ_{i1} + dQ_{i2}$$

In channel air flow:

$$dQ_{i1} = m_{air\ 1}C_{p_{air\ 1}}(T_{air\ 1\ i})(T_{air\ 1\ i} - T_{air\ 1\ i+1})$$

Out channel air flow:

$$dQ_{i2} = m_{air\ 2}C_{p_{air\ 2}}(T_{air\ 2\ i})(T_{air\ 2\ i} - T_{air\ 2\ i+1})$$

In natural convection air flow rate is difficult parameter, Velocity profile can be approximated as:

$$U_{air} = \sqrt{2gdz\beta(T_s - T_{air})}$$

EXPERIMENTAL MEASUREMENTS

Radiator thermal performance measurements are carried out according to EN 442-2 standard for testing radiators and convectors. According to this standard, Measurements are carried out for three different temperature zones

$$\Delta T = T_m - T_{room} = (30\ \mp 2.5)K$$

$$\Delta T = T_m - T_{room} = (50\ \mp 2.5)K$$

$$\Delta T = T_m - T_{room} = (60\ \mp 2.5)K$$

Where T_m is the arithmetic average temperature between inlet and exit of water

$$T_m = \frac{T_{w\_in} + T_{w\_out}}{2}$$

And T_{room} is the room temperatures. Room temperature and experiment wall temperatures should be set to a constant temperature of 20 ℃. In order to carry out this test, a laboratuary design with the specification of standards is required. Test results will be fit into a simple curvefitting equation in the form of

$$Q = \dot{m}\left(h_{w\_in} - h_{w\_out}\right) = K_M\Delta T^n$$

Where Q is the heat transfer, $\dot{m}$ is the mass flow rate of water flowing through radiator, h_w is the water enthalpies at inlet and outlet. K_M and n are the cırve fitting coefficients obtained as a result of experiments. In order to reduce measurements uncertainities, measuremnts of each point should be carried out several times (minimum of three times). A laboratory system according to EN 442-2 is developed and a wide range of radiators are measured by using this facility. Some of the measurement results and curve fitting coefficients are given below.

No	Sample	75/65°, DT=50K, 20 °C room Measured Thermal output		90/70°, DT=60K, 20 °C room Measured Thermal output		curve fitting coefficient n	Model Constant K_m
		Watt	Kcal/h	Watt	Kcal/h		
1	-PK, size (mm) 300x1000	562	483	711	612	1.302	3.442738

2	-PK, size (mm) 400x1000	722	621	916	788	1.296	4.542272
3	-PK, size (mm) 500x1000	876	753	1108	953	1.289	5.656291
4	-PK, size (mm) 600x1000	1026	882	1296	1115	1.2802	6.858
5	-PK, size (mm) 700x1000	1151	990	1454	1251	1.277	7.795927
6	-PK, size (mm) 800x1000	1280	1101	1615	1389	1.27	8.90958
7	-PK, size (mm) 900x1000	1399	1203	1777	1529	1.3047	8.5079
8	-PKP, size (mm) 300x1000	781	672	994	855	1.33	4.290071
9	-PKP, size (mm) 400x1000	998	858	1267	1090	1.321	5.6744
10	-PKP, size (mm) 500x1000	1193	1026	1517	1305	1.313	7.018701
11	-PKP, size (mm) 600x1000	1389	1194	1761	1515	1.3013	8.5478
12	-PKP, size (mm) 700x1000	1542	1326	1953	1680	1.295	9.725641
13	-PKP, size (mm) 800x1000	1699	1461	2147	1847	1.286	11.09334
14	-PKP, size (mm) 900x1000	1835	1578	2339	2013	1.3267	10.2333
15	-PKKP, size (mm) 300x1000	1001	861	1275	1097	1.321	5.708583
16	-PKKP, size (mm) 400x1000	1273	1095	1618	1392	1.319	7.303746
17	-PKKP, size (mm) 500x1000	1528	1314	1941	1670	1.317	8.83681
18	-PKKP, size (mm) 600x1000	1788	1537	2276	1958	1.3237	10.0782
19	-PKKP, size (mm) 700x1000	2006	1725	2550	2194	1.313	11.79777
20	-PKKP, size (mm) 800x1000	2233	1920	2835	2439	1.312	13.17181
21	-PKKP, size (mm) 900x1000	2452	2109	3112	2678	1.31	14.57782
22	-PKKPKP, size (mm) 300x1000	1448	1245	1846	1589	1.329	8.000871
23	-PKKPKP, size (mm) 400x1000	1810	1557	2305	1983	1.329	9.988665
24	-PKKPKP, size (mm) 500x1000	2149	1848	2737	2355	1.33	11.8142
25	-PKKPKP, size (mm) 600x1000	2486	2138	3171	2728	1.335	13.4073
26	-PKKPKP, size (mm) 700x1000	2791	2400	3556	3060	1.331	15.28534
27	-PKKPKP, size (mm) 800x1000	3091	2658	3939	3389	1.331	16.92893
28	-PKKPKP, size (mm) 900x1000	3391	2916	4340	3734	1.3483	17.3771

In the figures below a radiator measured in the lab is shown.

PROGRAM DEVELOPMENT & RESULTS

Computer codes in Java programming language is developed to calculate thermal performance of radiators. In order to calculate thermodynamic and thermophysical properties of air and water, equation of state programs are developed, and then finite difference model of radiator gheat transfer is developed. The computer classes used in this simulations are as follows:

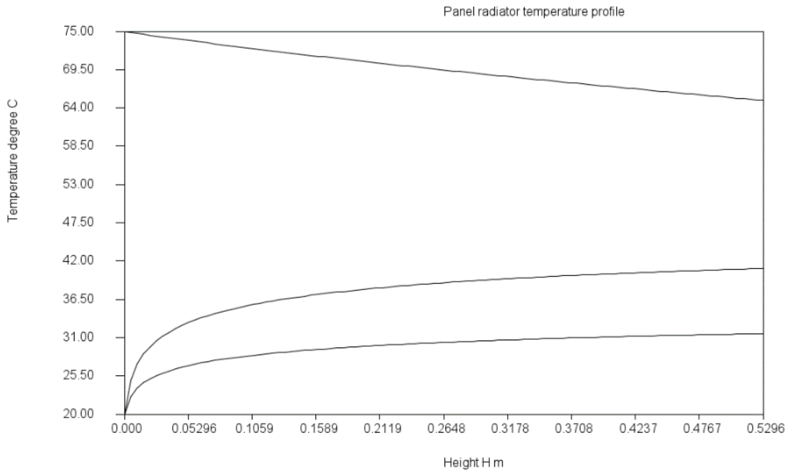
Class name	
steamIAPWS_IF97	Steam-water equation of state and thermophysical properties
air_PG_CS	air equation of state (perfect gas)
HT_radiator_elba1A	Finite difference heat transfer and heat exchanger simulation

Heat transfer predicitons from curve fitting function and computer model are shown below

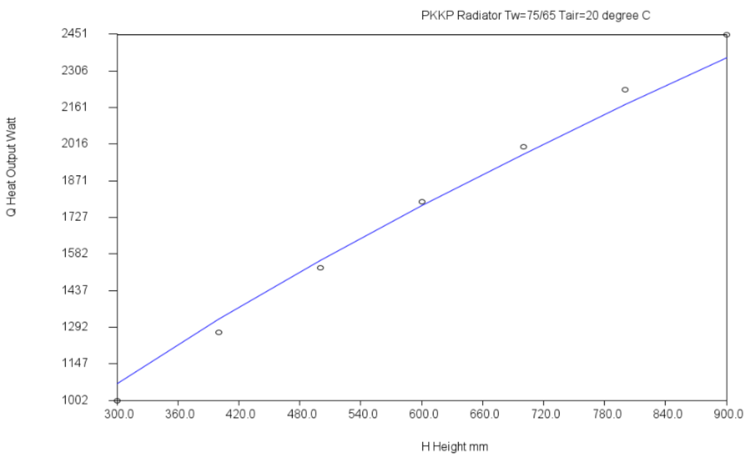
Calculated from curve fitting equation $Q = K_M \Delta T^n$ $T_w = 75/65\ ℃$ $T_{air} = 20\ ℃$	Calculated from curve fitting equation $Q = K_M \Delta T^n$ $T_w = 90/70\ ℃$ $T_{air} = 20\ ℃$	Calculated from simulation $T_w = 75/65\ ℃$ $T_{air} = 20\ ℃$	Calculated from simulation $T_w = 90/70\ ℃$ $T_{air} = 20\ ℃$

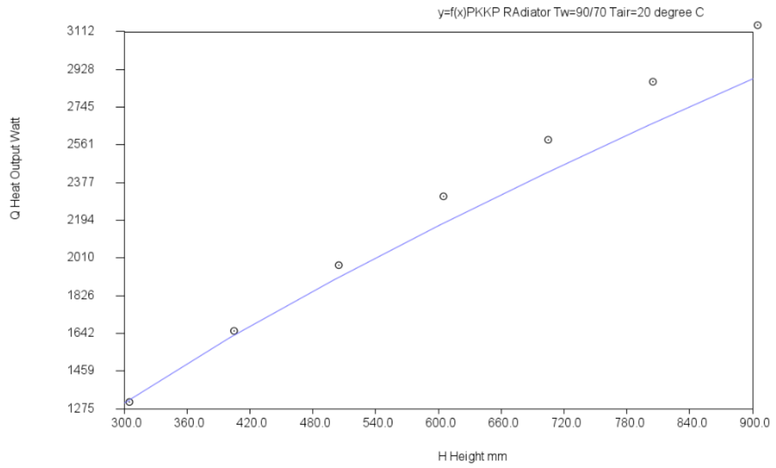
1002	1274.87072	1068	1303
1272	1617.80873	1324	1618
1527	1941.42554	1557	1902
1787.761446	2275.73549	1773	2168
2007	2549.83659	1977	2418
2232	2835.17575	2172	2656
2451	3112.22326	2359	2883

Temperature profile for water inside channels, and air through fins are given below.



Resul listed as a table, are also shown in graphic format in the following plots.





If we look at the model outputs, for 75/65 water temperature and T=20 ℃ air temperature, The results are relatively similar to curve fitting results based on experiments. For 90/70 water temperature and T=20 ℃ air temperature, model underpredict. The model still will be an imporant tool to predict raditor performance and paremeter changes and their effects. Model used here has a specific geometry of 2 panels and 2 fin structures as shown in the figure previously.

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