SIMULATION OF NATURAL CONVECTION FIN AND TUBE HEAT EXCHANGER FOR DOMESTIC HEATING APPLICATIONS

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1. INTRODUCTION

The design of a fin and tube heat exchanger attempts to compensate for the poor heat transfer coefficient on the air side by increasing the air side heat transfer area using fins. Fin and Tube type heat exchangers are usually used as forced convective heat transfer equipment for heating and cooling applications. They are not used commonly in natural convection type heating applications. In this study, fin and tube type natural convection heat exchanger thermal performance will be investigated by using simulation model and experimental measurements. Water flow inside tubes. Tubes are made of copper, Fins are made of alimininium and they are in plane configuration. Experimental measurements are carried out according to EN 442-2 standard for testing radiators and convectors. For computer simulations, a finite difference model in java computer language is developed.
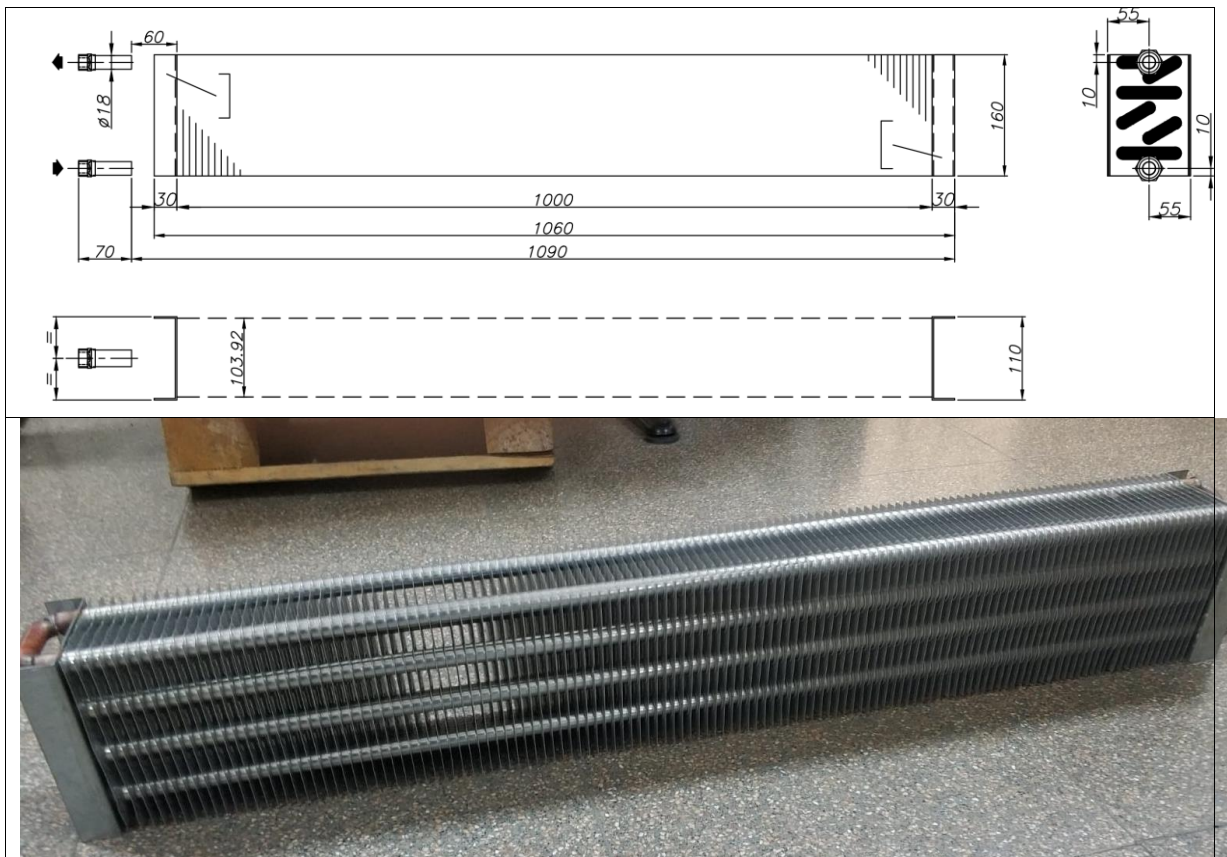

Figure 1. Fin and Tube natural convection heat exchanger used in experimental and computer simulation

2. COMPUTER SIMULATION MODELS

2.1 EQUATION OF STATES

In order to simulate the heat exchanger, equation of states and thermophysical properties such as viscosity, thermal convectivity is required. In fin and tube heat exchanger, water flows inside copper tubes, and air rise from outside through fins and tubes through natural convective and radiative heating. Therefore in order to developed a heat exchanger model, first requirement is to obtain properties of air and water

2.1.1 Thermodynamic & thermophysical properties air as a perfect gas

If an equation of state is given then all thermodynamic properties can be calculated by Consider an equation of state in the form of P(T,V)

$$ds = \left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv$$

where $\quad v = \frac{V}{N} \quad s = \frac{S}{N} \quad$ can be written.
If equation

$$C_v = C_v = T\left(\frac{\partial s}{\partial T}\right)_V$$

and Maxwel relation eqn

$$\left(\frac{\partial S}{\partial V}\right)_{T,N} = \left(\frac{\partial P}{\partial T}\right)_{V,N}$$

is used, equation becomes

$$ds = \frac{C_v(T)}{T} dT + \left(\frac{\partial P(T,v)}{\partial T}\right)_v dv$$

$$s = s_0 + \int_{T_0}^{T} \frac{C_v(T)}{T} dT + \int_{v_0}^{v} \left(\frac{\partial P(T,v)}{\partial T}\right)_v dv \quad (1.53a)$$

u equation of state 1.14 rewritten as $\quad du = Tds - Pdv$ and above equation is substituted for ds

$$du = T\left(\frac{C_v}{T} dT + \left(\frac{\partial P(T,v)}{\partial T}\right)_v dv\right) - P(T,v)dv$$

$$du = C_v(T)dT + \left(T\left(\frac{\partial P(T,v)}{\partial T}\right)_v - P(T,v)\right)dv$$

integration of the equation gives

$$u = u_0 + \int_{T_0}^{T} C_v(T)dT + \int_{v_0}^{v} \left(T\left(\frac{\partial P(T,v)}{\partial T}\right)_v - P(T,v)\right)dv$$

Ideal gas equation of state:

$$P(T,V) = \frac{NRT}{V}$$

$$v = \frac{V}{N}$$

$$P(T,v) = \frac{RT}{v}$$

Where R=8.3145 kJ/(kmolK) is gas constant

$$s = s_0 + \int_{T_0}^{T} \frac{C_v(T)}{T} dT + \int_{v_0}^{v} \left(\frac{\partial P(T,v)}{\partial T}\right)_v dv$$

$$\left(\frac{\partial P(T,v)}{\partial T}\right) = \frac{R}{V}$$

$$s = s_0 + \int_{T_0}^{T} \frac{C_v(T)}{T} dT + \int_{v_0}^{v} \frac{R}{V} dv$$

$$s = s_0 + \int_{T_0}^{T} \frac{C_v(T)}{T} dT + R ln \frac{v}{v_0}$$

$$s = s_0 + \int_{T_0}^{T} \frac{C_p(T)}{T} dT - R ln \frac{P}{P_0}$$

$$u = u_0 + \int_{T_0}^{T} C_v(T)dT + \int_{v_0}^{v} \left(T \left(\frac{\partial P(T,v)}{\partial T} \right)_v - P(T,v) \right) dv$$

$$u = u_0 + \int_{T_0}^{T} C_v(T)dT + \int_{v_0}^{v} \left(T \left(\frac{R}{v} \right)_v - \frac{RT}{v} \right) dv$$

$$u = u_0 + \int_{T_0}^{T} C_v(T)dT$$

In order to create perfect gas thermodynamic properties of air, The first step is look Specific heat data for air the standart air mixture

Nitrogen	N_2	7.808400E+01	0.781121204
Oxygen	O_2	2.094600E+01	0.209535433
Argon	Ar	9.340000E-01	0.009343364
Carbondioxide	CO_2	3.970000E-02	
Neon	Ne	1.818000E-03	
Helium	He	5.240000E-04	
Methane	CH_4	1.790000E-04	
Water vapor	H_2O	0.00001-0.05	

9.996400E+01   1

If it is assumed that Air is made of only N_2, O_2 and Ar, it will be changed to:

Name	Formula		% vol	M
Nitrogen	N2	78.084	0.781121204	28.014
Oxygen	O2	20.946	0.209535433	31.998
Argon	Ar	0.934	0.009343364	39.948
Air			1	28.96029

By finding specific heat data, we can able to establish the perfect gas equation of state. We will use Janaf tables from NIST (National Institute of Standards and Technology **Janaf.nist.gov)** to obtain specific heat data**.** After obtaining Specific heat data for Nitrogen, Oxygen and Argonne, Specific heat data for air is obtained by using ideal gas mixing rules. After the airspecific heat data is obtainedCubic spline interpolation will be applied to model air properties.

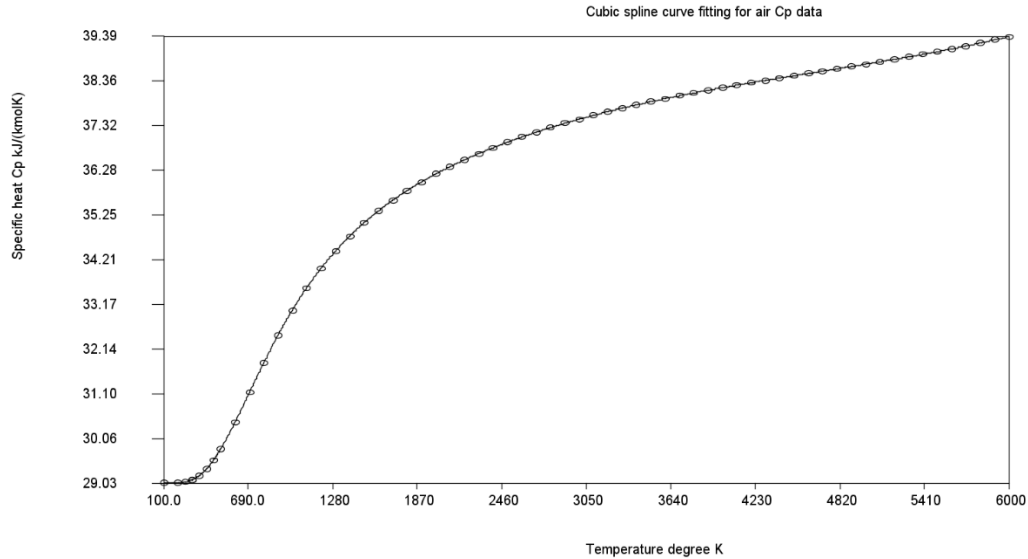If a third degree polinomial is considered:

$$r_k(x) = a_k(x - x_k)^3 + b_k(x - x_k)^2 + c(x - x_k)^3 + y_k \quad 1 \le k \le n \quad (2.1.18)$$

In the interpolation proses polinoms should be passing through all data points

$$r_k(x_{k+1}) = y_{k+1} \quad 1 \le k \le n \quad (2.1.19)$$

In the same time the first derivative of the polynomial should also be continious while passing from one polynomial to the next one at the data point

$$r'_{k-1}(x_k) = r'_k(x_k) \quad 1 \le k \le n \quad (2.1.20)$$

For the third degree polinomial second derivative of the polynomial should also be continious while passing from one polynomial to the next one at the data point
$$r''_{k-1}(x_k) = r''_k(x_k) \quad 1 \le k \le n \quad (2.1.21)$$
All these conditions are not enough to solve the coefficients of the polinomials. Two more conditions are required. This two additional conditions (A and B of the following equation) can be given by user
$$r''_1(x_1) = A \qquad r''_{n-1}(x_n) = B \quad (2.1.22)$$
They are the second derivatives at the both hand of the series of polinomials. If A and B values are taken equals to 0, it is called a **natural cubic spline**. Other end conditions such as the ones depends one the first derivatives can also be set to solve the system of equations.
Defining $h_k = x_{k+1} - x_k \quad 1 \le k \le n \quad (2.1.23)$
System of equations become:
$$a_k h_k^3 + b_k h_k^2 + b_k h_k = y_{k+1} - y_k \quad 1 \le k \le n \quad (2.1.24)$$

$$3a_{k-1}h_{k-1}^2 + 2b_{k-1}h_{k-1} + c_{k-1} - c_k = 0$$

$$6a_{k-1}h_{k-1} + 2b_{k-1} + 2b_k = 0$$

$$3b_0 = 0$$

$$6a_{n-1}h_{n-1} + 2b_{n-1} = 0$$

This set contains 3n-3 equations. This could a considerable load to the system of equation solving programs. To make calculation load simpler a special third degree polinomial can be considered. If our cubic polinomial is in the form of:
$$s_k(x) = a_k(x - x_k) + b_k(x_{k+1} - x) + [(x - x_k)^3 c_{k+1} + (x_{k+1} - x)^3 c_k]/(6h_k) \quad 1 \le k \le n \quad (2.1.25)$$
then derivative equations becomes
$$s'_k(x) = a_k - b_k + [(x - x_k)^2 c_{k+1} - (x_{k+1} - x)^2 c_k]/h_k \quad 1 \le k \le n \quad (2.1.26)$$

$$s''_k(x) = [(x - x_k)c_{k+1} - (x_{k+1} - x)c_k]/h_k \qquad 1 \le k \le n$$

a_k ve b_k coefficients can be expressed as a function of c_k
$$b_k = \frac{[6y_k - h_k c_k]}{6h_k} \quad 1 \le k \le n \quad (2.1.27)$$
$$a_k = \frac{[6y_{k+1} - h_k^2 c_{k+1}]}{6h_k} \quad 1 \le k \le n \quad (2.1.28)$$
In this case only c_k terms left in the system of equations to be solved.
$$h_{k-1}c_{k-1} + 2(h_{k-1} - h_k)c_{k-1} + h_k c_{k+1}c_{k+1} = 6\left[\frac{y_{k+1} - y_k}{h_k} - \frac{y_k - y_{k-1}}{h_{k-1}}\right] \quad 1 \le k \le n \quad (2.1.29)$$
This system of equation has only n-2 terms to be solved. By making definition
$$w_k = \frac{y_{k+1} - y_k}{h_k}, \quad 1 \le k \le n \quad (2.1.31)$$
System of equation becomes

$$
\begin{bmatrix}
1 & & & & & & \\
h_1 & 2(h_1 + h_2) & h_2 & & & & \\
 & h_2 & 2(h_2 + h_3) & & & & \\
 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
 & & & \cdots & 2(h_{n-3} + h_{n-2}) & h_{n-2} & \\
 & & & & h_{n-2} & 2(h_{n-2} + h_{n-1}) & h_{n-1} \\
 & & & & & & 1
\end{bmatrix}
\begin{Bmatrix}
c_0 \\ c_1 \\ c_2 \\ \vdots \\ c_{n-2} \\ c_{n-1} \\ c_n
\end{Bmatrix}
=
\begin{Bmatrix}
A \\ 6(w_2 - w_1) \\ 6(w_3 - w_2) \\ \vdots \\ 6(w_{n-2} - w_{n-3}) \\ 6(w_{n-1} - w_{n-2}) \\ B
\end{Bmatrix}
\quad (2.1.30)
$$

Where A and B are the second derivative end conditions. A and B should be defined by user. Another important property of the above matrix is that it is a band matrix, therefore less amount of calculation is required to solve it (by using band matrix algorithms such as Thomas algorithm).

Cubic spline method has two advantages, the first is very accurate represantion of data, and the second one is ability to directly integrate and derivate the spline function.

Cubic spline curve fitting for air Cp data

Formulations of other thermophysical and thermodynamic properties

In order to calculate thermopysical properties (thermal conductivity and viscosity) of dry air Kadoya et al[135] equations are used. This equations has the following form:

$$\eta_0(T_r) = A_0 T_r + A_1 T_r^{0.5} + A_2 + \frac{A_3}{T_r} + \frac{A_4}{T_r^2} + \frac{A_5}{T_r^3} + \frac{A_6}{T_r^4}$$

$$\Delta\eta(\rho_r) = \sum_{i=1}^{4} B_i \rho_r^i$$

$$\eta(T_r, \rho_r) = H[\eta_0(T_r) + \Delta\eta(\rho_r)]$$

$$k_0(T_r) = C_0 T_r + C_1 T_r^{0.5} + C_2 + \frac{C_3}{T_r} + \frac{C_4}{T_r^2} + \frac{C_5}{T_r^3} + \frac{C}{T_r^4}$$

$$\Delta k(\rho_r) = \sum_{i=1}^{4} D_i \rho_r^i$$

$$k(T_r, \rho_r) = \Lambda[k_0(T_r) + \Delta k(\rho_r)]$$

Where $\rho_r = \rho/\rho^*$ $T_r = T/T^*$

Coefficients of the equations are given in Table

Table Coefficients of viscosity and thermal conductivity equations

$T^* = 132.5$ K	$\rho^* = 314.3$ kg/m3	$\Lambda = 25.9778$ $(10^{-3} W/(mK)$	H=6.1609 $(10^{-6}$ Pas)	
i	A$_i$	B$_i$	C$_i$	D$_i$
0	0.128517	0.465601	0.239503	0.402287
1	2.60661	1.26469	0.00649768	0.356603
2	-1	-0.511425	1	-0.163159
3	-0.709661	0.2746	-1.92615	0.138059
4	0.662534		2.00383	-0.0201725
5	-0.197846		-1.07553	
6	0.00770147		0.229414	

2.1.2 Thermodynamic & thermophysical properties of water and steam

In recent years maximum operating temperatures and pressures of Rankine cycle power plants has increased. International Association for the Properties of Water and Steam(IAPWS) is developed a new set of equation of states which are more accurate and covers larger range of data. This new set of equations are developed in 1997[59]. Steam properties are given by 5 sets of equation of states, as shown in the Figure
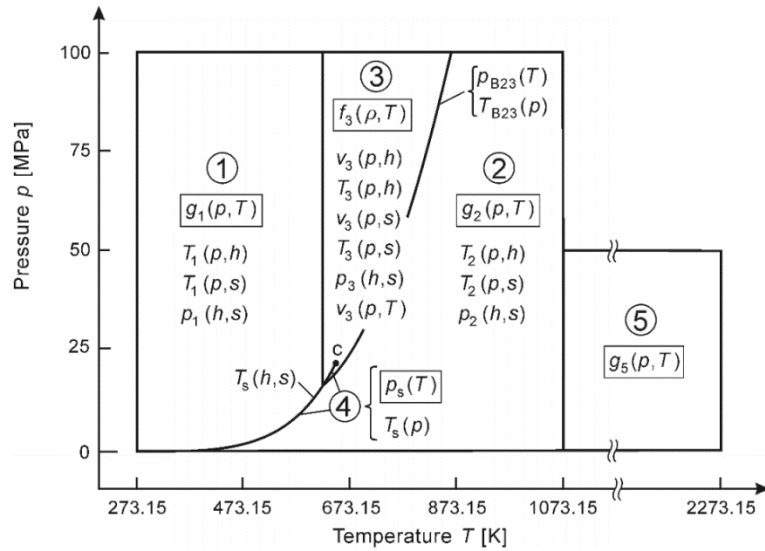
Figure IAPWS 97 Equation of state regions for steam

The first equation, which covers basically liquid region has the following gibbs free energy form:

$$\frac{g_1(P,T)}{RT} = \gamma(\pi,\tau) = \sum_{i=1}^{34} n_i(71-\pi)^{I_i}(\tau-1222)^{J_i}$$

Where $\pi = \frac{P}{P^*}$ $\tau = \frac{T^*}{T}$ $p^* = 16.62$ MPa and $T^* = 1386$ K   R=0461526 kJ/(kgK)

Table 2.6.4 coefficients of eqn. 2.6.11

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	0	-2	0.14632971213167	18	2	3	-4.4141845331E-06
2	0	-1	-0.84548187169114	19	2	17	-7.2694996298E-16
3	0	0	-3.75636036720400	20	3	-4	-3.1679644845E-05
4	0	1	3.38551691683850	21	3	0	-2.8270797985E-06
5	0	2	-0.95791963387872	22	3	6	-8.5205128120E-10
6	0	3	0.15772038513228	23	4	-5	-2.2425281908E-06
7	0	4	-0.01661641719950	24	4	-2	-6.5171222896E-07
8	0	5	0.00081214629984	25	4	10	-1.4341729938E-13
9	1	-9	0.00028319080124	26	5	-8	-4.0516996860E-07
10	1	-7	-0.00060706301566	27	8	-11	-1.2734301742E-09
11	1	-1	-0.01899006821842	28	8	-6	-1.7424871231E-10
12	1	0	-0.03252974877051	29	21	-29	-6.8762131296E-19
13	1	1	-0.02184171717541	30	23	-31	1.4478307829E-20
14	1	3	-0.00005283835797	31	29	-38	2.6335781663E-23
15	2	-3	-0.00047184321073	32	30	-39	-1.1947622640E-23
16	2	0	-0.00030001780793	33	31	-40	1.8228094581E-24
17	2	1	0.00004766139391	34	32	-41	-9.3537087292E-26

Thermodynamic relations can be calculated from these thermodynamic relations

Specific volume: $v = \left(\frac{\partial g}{\partial P}\right)_T$ (2.6.12)

Specific enthalpy: $h = g - T\left(\frac{\partial g}{\partial T}\right)_P$ (2.6.13)

Specific internal energy: $u = g - T\left(\frac{\partial g}{\partial T}\right)_P - P\left(\frac{\partial g}{\partial P}\right)_T$ (2.6.14)

Specific entropy: $s = \left(\frac{\partial g}{\partial T}\right)_P$ (2.6.15)

Specific isobaric heat capacity: $C_p = \left(\frac{\partial h}{\partial T}\right)_P$ (2.6.16)

Specific isochoric heat capacity: $C_v = \left(\frac{\partial u}{\partial T}\right)_v$ (2.6.17)

The second equation equation, which covers vapor region has the following gibbs free energy form:

$$\frac{g_2(P,T)}{RT} = \gamma(\pi,\tau) = \gamma^0(\pi,\tau) + \gamma^r(\pi,\tau) \quad (2.6.18)$$

Where $\pi = \frac{P}{P^*}$ $\tau = \frac{T^*}{T}$ R=0.461526 kJ/(kgK), $\gamma^0(\pi,\tau)$ is the ideal gas part of EOS, and $\gamma^r(\pi,\tau)$ is the real gas departure the EOS. İdeal gas part equation:

$$\gamma^0(\pi,\tau) = \ln(\pi) + \sum_{i=1}^{9} n_i^0 \tau^{J_i} \quad (2.6.19)$$

Where P*=1MPa and T*=540 K

Table 2.6.5 coefficients of eqn. 2.6.19

i	J_i	n_i^0	i	J_i	n_i^0
1	0	-9.692768650E+00	6	-2	1.4240819171E+00
2	1	1.008665597E+01	7	-1	-4.3839511319E+00
3	-5	-5.608791128E-03	8	2	-2.8408632461E-01
4	-4	7.145273808E-02	9	3	2.1268463753E-02
5	-3	-4.071049822E-01			

dimensionless residual part of the basic equation $g2(p,T)$ is as follows:

$$\gamma^r(\pi,\tau) = \sum_{i=1}^{43} n_i \pi^{I_i}(\tau - 0.5)^{J_i} \quad (2.6.20)$$

Where P*=1MPa and T*=540 K

Table 2.6.6 coefficients of eqn. 2.6.20

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	1	0		7		0	-5.9059564324270E-18
2	1	1	-1.7834862292358E-02	24	7	11	-1.2621808899101E-06
3	1	2	-4.5996013696365E-02	25	7	25	-3.8946842435739E-02
4	1	3	-5.7581259083432E-02	26	8	8	1.1256211360459E-11
5	1	6	-5.0325278727930E-02	27	8	36	-8.2311340897998E+00
6	2	1	-3.3032641670203E-05	28	9	13	1.9809712802088E-08
7	2	2	-1.8948987516315E-04	29	10	4	1.0406965210174E-19
8	2	4	-3.9392777243355E-03	30	10	10	-1.0234747095929E-13
9	2	7	-4.3797295650573E-02	31	10	14	-1.0018179379511E-09
10	2	36	-2.6674547914087E-05	32	16	29	-8.0882908646985E-11
11	3	0	2.0481737692309E-08	33	16	50	1.0693031879409E-01
12	3	1	4.3870667284435E-07	34	18	57	-3.3662250574171E-01
13	3	3	-3.2277677238570E-05	35	20	20	8.9185845355421E-25
14	3	6	-1.5033924542148E-03	36	20	35	3.0629316876232E-13
15	3	35	-4.0668253562649E-02	37	20	48	-4.2002467698208E-06
16	4	1	-7.8847309559367E-10	38	21	21	-5.9056029685639E-26
17	4	2	1.2790717852285E-08	39	22	53	3.7826947613457E-06
18	4	3	4.8225372718507E-07	40	23	39	-1.2768608934681E-15
19	5	7	2.2922076337661E-06	41	24	26	7.3087610595061E-29
20	6	3	-1.6714766451061E-11	42	24	40	5.5414715350778E-17
21	6	16	-2.1171472321355E-03	43	24	58	-9.4369707241210E-07
22	6	35	-2.3895741934104E+01				

Region 3 equation is given as Helmholts free energy form:

$$\frac{f_3(\rho,T)}{RT} = \phi(\delta,\tau) = n_1 \ln(\delta) + \sum_{i=2}^{40} n_i \delta^{I_i} \tau^{J_i} \quad (2.6.21)$$

Where $\delta = \frac{\rho}{\rho^*}$ $\tau = \frac{T^*}{T}$, T*=T_c=647.096 and R=0461526 kJ/(kgK)

Table 2.6.4 coefficients of eqn. 2.6.11

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	0	0	1.065807002851E+00	21	3	4	-2.0189915023570E+00
2	0	0	-1.573284529024E+01	22	3	16	-8.2147637173963E-03
3	0	1	2.094439697431E+01	23	3	26	-4.7596035734923E-01
4	0	2	-7.686770787872E+00	24	4	0	4.3984074473500E-02
5	0	7	2.618594778795E+00	25	4	2	-4.4476435428739E-01
6	0	10	-2.808078114862E+00	26	4	4	9.0572070719733E-01
7	0	12	1.205336969652E+00	27	4	26	7.0522450087967E-01
8	0	23	-8.456681281250E-03	28	5	1	1.0770512626332E-01
9	1	2	-1.265431547771E+00	29	5	3	-3.2913623258954E-01
10	1	6	-1.152440780668E+00	30	5	26	-5.0871062041158E-01

11	1	15	8.852104398432E-01	31	6	0	-2.2175400873096E-02
12	1	17	-6.420776518161E-01	32	6	2	9.4260751665092E-02
13	2	0	3.849346018667E-01	33	6	26	1.6436278447961E-01
14	2	2	-8.521470882421E-01	34	7	2	-1.3503372241348E-02
15	2	6	4.897228154188E+00	35	8	26	-1.4834345352472E-02
16	2	7	-3.050261725697E+00	36	9	2	5.7922953628084E-04
17	2	22	3.942053687915E-02	37	9	26	3.2308904703711E-03
18	2	26	1.255840842431E-01	38	10	0	8.0964802996215E-05
19	3	0	-2.799932969871E-01	39	10	1	-1.6557679795037E-04
20	3	2	1.389979956946E+00	40	11	26	-4.4923899061815E-05

It should be noted that this set of equation is function of density and temperature, and basic equation is helmholts equation so, let us list definition of other thermodynamic properties

Pressure: $P = \rho^2 \left(\frac{\partial f}{\partial \rho}\right)_T$

Specific enthalpy: $h = f - T \left(\frac{\partial f}{\partial T}\right)_p + \rho \left(\frac{\partial f}{\partial \rho}\right)_T$

Specific internal energy: $u = f - T \left(\frac{\partial f}{\partial T}\right)_p$

Specific entropy: $s = \left(\frac{\partial f}{\partial T}\right)_\rho$

Specific isobaric heat capacity: $C_p = \left(\frac{\partial h}{\partial T}\right)_p$

Specific isochoric heat capacity: $C_v = \left(\frac{\partial u}{\partial T}\right)_v$

Region 4 of the equation defines saturation region. The basic equation is given as a polynomial

$$\beta^2 \vartheta^2 + n_1 \beta^2 \vartheta + n_2 \beta^2 + n_3 \beta \vartheta^2 + n_4 \beta \vartheta + n_5 \beta + n_6 \vartheta^2 + n_7 \vartheta + n_8 = 0 \quad (2.6.18)$$

Where

$$\beta = \left(\frac{P_S}{P^*}\right)^{0.25}$$
$$\vartheta = \frac{T_S}{T^*} + \frac{n_9}{\left(\frac{T_S}{T^*}\right) - n_{10}}$$

From this equation both saturation pressure and saturation temperature equation can be derived.

$$\frac{P_S}{P^*} = \left[\frac{2C}{-B + (B^2 - 4AC)^{0.5}}\right]^4$$

Where P*=1 MPa

$A = \vartheta^2 + n_1 \vartheta + n_2$
$B = n_3 \vartheta^2 + n_4 \vartheta + n_5$
$C = n_6 \vartheta^2 + n_7 \vartheta + n_8$

Table coefficients of eqn

i	n_i	i	n_i
1	1.1670521453E+03	6	1.4915108614E+01
2	-7.2421316703E+05	7	-4.8232657362E+03
3	-1.7073846940E+01	8	4.0511340542E+05
4	1.2020824702E+04	9	-2.3855557568E-01
5	-3.2325550322E+06	10	6.5017534845E+02

It is also possible to drive saturation temperature equation from the basic polynomial as:

$$\frac{T_S}{T^*} = \frac{n_{10} + D - [(n_{10} + D)^2 - 4(n_9 + n_{10}D)]^{0.5}}{2} \quad (2.6.22)$$

Where T*=1 K

$$D = \frac{2G}{-F - (F^2 - 4EG)^{0.5}}$$
$$E = \beta^2 + n_3 \beta + n_6$$
$$F = n_1 \beta^2 + n_4 \beta + n_7$$
$$G = n_2 \beta^2 + n_5 \beta + n_8$$

And the final region for steam is region 5, again given as gibbs free equation type EOS

$$\frac{g_5(P,T)}{RT} = \gamma(\pi, \tau) = \gamma^0(\pi, \tau) + \gamma^r(\pi, \tau) \quad (2.6.23)$$

Where $\pi = \frac{P}{P^*}$ $\tau = \frac{T^*}{T}$ R=0.461526 kJ/(kgK), $\gamma^0(\pi, \tau)$ is the ideal gas part of EOS, and $\gamma^r(\pi, \tau)$ is the real gas difference of the EOS. İdeal gas part equation:

$$\gamma^0(\pi, \tau) = \ln(\pi) + \sum_{i=1}^{9} n_i^0 \tau^{J_i} \quad (2.6.24)$$

Where p*=1MPa and T*=1000 K

Table coefficients of eqn.

i	J_i^0	n_i^0	i	J_i^0	n_i^0
1	0	-13.1799836742	4	-2	0.3690153498
2	1	6.8540841634	5	-1	-3.1161318214
3	-3	-0.0248051489	6	2	-0.3296162654

The real gas part of the equation

$$\gamma^r(\pi, \tau) = \sum_{i=1}^{43} n_i \pi^{I_i} \tau^{J_i} \quad (2.6.25)$$

Table coefficients of eqn. 2.6.25

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	1	1	1.5736404855E-03	4	2	3	2.2440037409E-06
2	1	2	9.0153761674E-04	5	2	9	-4.1163275453E-06
3	1	3	-5.0270077678E-03	6	3	7	3.7919454823E-08

Formulations of other thermophysical and thermodynamic properties

In order to calculate thermopysical properties (thermal conductivity and viscosity) of dry air Kadoya et al[135] equations are used. This equations has the following form:

$$\eta_0(T_r) = A_0 T_r + A_1 T_r^{0.5} + A_2 + \frac{A_3}{T_r} + \frac{A_4}{T_r^2} + \frac{A_5}{T_r^3} + \frac{A_6}{T_r^4}$$

$$\Delta\eta(\rho_r) = \sum_{i=1}^{4} B_i \rho_r^i$$

$$\eta(T_r, \rho_r) = H[\eta_0(T_r) + \Delta\eta(\rho_r)]$$

$$k_0(T_r) = C_0 T_r + C_1 T_r^{0.5} + C_2 + \frac{C_3}{T_r} + \frac{C_4}{T_r^2} + \frac{C_5}{T_r^3} + \frac{C}{T_r^4}$$

$$\Delta k(\rho_r) = \sum_{i=1}^{4} D_i \rho_r^i$$

$$k(T_r, \rho_r) = \Lambda[k_0(T_r) + \Delta k(\rho_r)]$$

Where $\rho_r = \rho/\rho^* \quad T_r = T/T^*$

Coefficients of the equations are given in Table

Table Coefficients of viscosity and thermal conductivity equations

$T^* = 132.5$ K	$\rho^* = 314.3$ kg/m3	$\Lambda = 25.9778 \ (10^{-3} W/(mK)$	H=6.1609 (10^{-6} Pas)	
i	A_i	B_i	C_i	D_i
0	0.128517	0.465601	0.239503	0.402287
1	2.60661	1.26469	0.00649768	0.356603
2	-1	-0.511425	1	-0.163159
3	-0.709661	0.2746	-1.92615	0.138059
4	0.662534		2.00383	-0.0201725
5	-0.197846		-1.07553	
6	0.00770147		0.229414	

Viscosity and thermal conductivity values of steam and water are taken from IAPWS Industrial Formulation 1997[15]. This equations are as follows:

Viscosity equations:

$\eta(\rho,T) = \psi(\delta, \theta) = \eta^* [\psi_0(\theta)\psi_1(\delta, \theta)]$

Where $\eta^* = 10^{-6} \ Pas \quad \delta = \frac{\rho}{\rho^*} \quad \theta = T/T^*$

with $T^* = T_c = 647.096 \ K \quad \rho^* = \rho_c = 322 \ kg/m^3$

$\psi_0(\theta) = \theta^{0.5}[\sum_{i=1}^{4} n_i^0 \theta^{1-i}]^{-1}$ Coefficients of equation given below:

Table 3.2 Coefficients of equation

i	n_i^0
1	0.167752e-1
2	0.220462e-1
3	0.6366564e-2
4	-0.241605e-2

$$\psi_1(\delta,\theta) = exp\left[\delta \sum_{i=1}^{21} n_i (\delta-1)^{I_i}\left(\frac{1}{\theta}-1\right)^{J_i}\right]$$

Table Coefficients of equation

i	I_i	J_i	N_i	i	I_i	J_i	N_i
1	0	0	5.200940E-01	12	2	2	-7.724790E-01
2	0	1	8.508950E-02	13	2	3	-4.898370E-01
3	0	2	-1.083740E+00	14	2	4	-2.570400E-01
4	0	3	-2.895550E-01	15	3	0	1.619130E-01
5	1	0	2.225310E-01	16	3	1	2.573990E-01
6	1	1	9.991150E-01	17	4	0	-3.253720E-02
7	1	2	1.887970E+00	18	4	3	6.984520E-02
8	1	3	1.266130E+00	19	5	4	8.721020E-03
9	1	5	1.205730E-01	20	6	3	-4.356730E-03
10	2	0	-2.813780E-01	21	6	5	-5.932640E-04
11	2	1	-9.068510E-01				

Thermal conductivity equations

$$\frac{k(\rho,T)}{\lambda^*} = \Lambda(\delta,\theta)=\Lambda_0(\theta)+\Lambda_1(\delta)+\Lambda_2(\delta,\theta)$$

$$\Lambda_0(\theta)=\theta^{0.5}\sum_{i=1}^{4} n_i^0 \theta^{i-1}$$

Table Coefficients of equation

i	n_i^0
1	0.102811e-1
2	0.299621e-1
3	0.156146e-1
4	-0.422464e-2

$$\Lambda_1(\delta)=n_1 + n_2\delta+n_3 exp[n_4(\delta + n_5)^2]$$

Table Coefficients of equation

i	n_i
1	0.39707
2	0.400302
3	-0.171587e4
4	-0.239219e1

$$\Lambda_2(\delta,\theta)=(n_1\theta^{-10}+n_2)\delta^{1.8}exp[n_2(1-\delta^{2.8})] + n_4 A\delta^B exp\left[\left(\frac{B}{1+B}\right)(1-\delta^{1+B})\right] + n_5 exp[n_6\theta^{1.5}+n_7\delta^{-5}]$$

$$A(\theta) = 2 + n_8(\Delta\theta)^{-0.6}$$

$$B(\theta) = \begin{cases} (\Delta\theta)^{-1} & for\ \theta \geq 1 \\ n_9(\Delta\theta)^{-0.6} & for\ \theta < 1 \end{cases} \quad 3.27b\ \ with\ \ \Delta\theta = |\theta-1| + n_{10}$$

Table Coefficients of equation

i	n_i	i	n_i
1	7.013090E-02	6	-4.117170E+00
2	1.185200E-02	7	-6.179370E+00
3	6.428570E-01	8	8.229940E-02
4	1.699370E-03	9	1.009320E+01
5	-1.020000E+00	10	3.089760E-03

2.2 HEAT TRANSFER EQUATIONS

Internal flow :

Water is flowing inside round tubes, Therefore internal flow heat transfer for water will be investigated.

2.2.1 One phase pressure drop

One phase pressure drop is usually calculates by using Colebrrok-White equations. This equation requires root finding, and an initial guess for the root finding process. Goudar-Sonnad equition which has the same type of accuracy with Colebrook-White equation but is not required any root finding process will be utilised in prossere drop calculations

Darcy-Weisbach pressure drop formula is given as:

$$\Delta P = f \frac{L}{D} \rho \frac{U^2}{2}$$

In this equation ΔP is the pressure drop, f is the friction factor, L is the length of pipe, U is fluid average velocity, D is pipe diameter. Friction factor f is depends on flow regime. For laminar flow ($Re_{critical}=2300$) Hagen-Poiseuille equation can be used.

$Re = \frac{\rho U D}{\mu}$ where Re is Reynolds' number, μ is dynamic viscosity, D pipe inlet diameter and ρ is the density.

$$f = \frac{64}{Re}$$

For turbulent region Colebrook-White(1937) [4] equation is existed.

$$\frac{1}{\sqrt{f}} = -2log_{10}\left[\frac{(\varepsilon/D)}{3.7} + \frac{2.51}{Re\sqrt{f}}\right]$$

In this equation ε is called surface roughness.

Goudar- Sonnad equation (2008) Valid region: all values

$$a = \frac{2}{\ln(10)}$$
$$b = \frac{(\varepsilon/D)}{3.7}$$
$$d = \frac{\ln(10)}{5.02}Re$$
$$s = bd + \ln\left(\frac{d}{q}\right);$$
$$q = s^{\left(\frac{s}{s+1}\right)}$$
$$g = bd + \ln\left(\frac{d}{q}\right)$$
$$z = \frac{q}{g}$$
$$\delta_{LA} = \frac{g}{g+1}z$$
$$\delta_{CFA} = \delta_{LA}\left(1 + \frac{z/2}{(g+1)^2+(\frac{z}{3})(2g-1)}\right)$$
$$\frac{1}{\sqrt{f}} = a\left[ln\left(\frac{d}{q}\right) + \delta_{CFA}\right]$$

2.2.2 inside tube heat transfer:

Laminar flow
$$Nu = 3.66$$
Heat transfer equations **Fully developed transitional/intermittent region** T_s=const
Abraham-Sparrow-Tong[34] equation

$$Nu = 2.2407\left(\frac{Re}{1000}\right)^4 - 29.499\left(\frac{Re}{1000}\right)^3 + 142.32\left(\frac{Re}{1000}\right)^2 - 292.51\left(\frac{Re}{1000}\right) + 219.88 \qquad 2300 \leq$$
$Re \leq 3100$

Abraham recommended Gnilenski equation to be used above Re>3100

Gnielinski[33] equation

$$Nu = \frac{\left(\frac{f}{8}\right)(Re-1000)Pr}{1.07+12.7\left(\frac{f}{8}\right)^{.5}(Pr^{\frac{2}{3}}-1)} \qquad 0.5 \leq Pr \leq 2000 \quad 2300 \leq Re \leq 5 10^6$$

2.2.3 External natural convection:

Churchill & Chu Equation for all Ra range[36] (valid both turbulent and laminar cases)

Rayleigh Number: $Ra_x = Gr_x Pr = \frac{g\beta(T_s-T_\infty)x^3}{v\alpha} \qquad \alpha = \frac{k}{\rho C_p}$

Critical Rayleigh Number $Ra_{x,critical} = 10^9$

$Nu_L = 4/3 Nu_x$

$$Nu_L = \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1+\left(\frac{0.492}{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2$$

Natural convection heat transfer in channels

For symmetrical heated, isothermal plates Elenbaas[58] equation is as folows:

$$Nu_S = \left(\frac{q/A}{T_s-T_\infty}\right)\frac{S}{k} = \frac{1}{24}Ra_S\left(\frac{S}{L}\right)\left[1-exp\left(-\frac{35}{Ra_S\left(\frac{S}{L}\right)}\right)\right]^{3/4}$$

where **Rayleigh Number:** $Ra_S = \frac{g\beta(T_s-T_\infty)S^3}{v\alpha} \qquad 10^{-1} \leq Ra_S \leq 10^5$
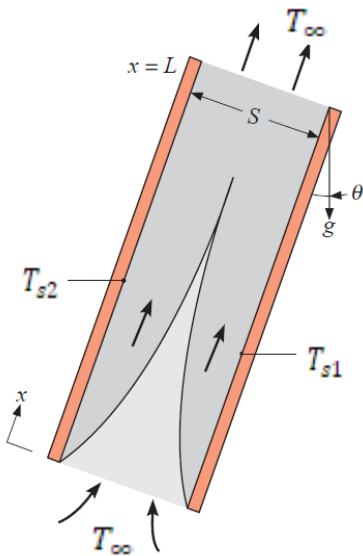
For constant heat flux cases

$$Nu_S = \left(\frac{q''_s}{T_s-T_\infty}\right)\frac{S}{k} \qquad Ra_S^* = \frac{g\beta q''_s S^4}{kv\alpha}$$

for symmetric fully developed constant heat flux

$Nu_{SL} = 0.144[Ra_S^*(S/L)]^{1/2}$

for asymmetric fully developed constant heat flux

$Nu_{SL} = 0.204[Ra_S^*(S/L)]^{1/2}$

Bar-Cohen-Rohsenow[59] equation:

For isothermal plates

$$\text{Nu}_{SL} = \left[\frac{576}{\left(Ra_S\left(\frac{S}{L}\right)^2\right)} + \frac{2.87}{\left(Ra_S\left(\frac{S}{L}\right)^{1/2}\right)}\right]^{-1/2} \quad 10 \leq \text{Ra}_S \leq 100 \quad T_{s1} = T_{s2} \text{ symmetric isothermal}$$

$$\text{Nu}_{SL} = \left[\frac{144}{\left(Ra_S\left(\frac{S}{L}\right)^2\right)} + \frac{2.87}{\left(Ra_S\left(\frac{S}{L}\right)^{1/2}\right)}\right]^{-1/2} \quad 10 \leq \text{Ra}_S \leq 100 \quad T_{s1}, \; q_{s2}'' = 0 \text{ isothermal adiabatic}$$

3. SIMULATION MODELLING

We will consider a single fin and a single pipe section. In this case fin will be exposed to natural convection. Pipe section will also exposed to natural convection, but different heat transfer equations will be used for pipe and fin. Furthermore for fins, fin efficiency should be taken into effect.
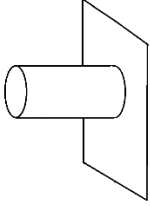
$$dz = \frac{L}{N}$$

Where L= pipe length
N= number of finite difference division

$$dQ_i = U_i dA_i (T_{water} - T_{air})$$

$$\frac{1}{U_i} = \frac{1}{h_{water\,i}} + \frac{t_{panel}}{k_{panel}} + \frac{1}{\eta_{fin\,i} h_{air\,i}}$$

$$M = \frac{2hl_{fin}}{kt_{fin}}$$

$$\eta_{fin} = \left(\frac{\tanh{(M)}}{M}\right)$$

$$dQ_i = m_{water} C_{p_{water}} (T_{water\,i})(T_{water\,i} - T_{water\,i+1})$$

In natural convection air flow rate is difficult parameter, Velocity profile can be approximated as:

$$U_{air} = \sqrt{2g\,dz\,\beta(T_s - T_{air})}$$

4. EXPERIMENTAL MEASUREMENTS

Fin and tube heat exchanger thermal performance measurements are carried out according to EN 442-2 standard for testing radiators and convectors. According to this standard, Measurements are carried out for three different temperature zones

$$\Delta T = T_m - T_{room} = (30 \mp 2.5)K$$

$$\Delta T = T_m - T_{room} = (50 \mp 2.5)K$$

$$\Delta T = T_m - T_{room} = (60 \mp 2.5)K$$

Where T_m is the arithmetic average temperature between inlet and exit of water

$$T_m = \frac{T_{w\_in} + T_{w\_out}}{2}$$

And T_{room} is the room temperatures. Room temperature and experiment wall temperatures should be set to a constant temperature of 20 °C. In order to carry out this test, a laboratuary design with the specification of standards is required. Test results will be fit into a simple curvefitting equation in the form of

$$Q = \dot{m}(h_{w\_in} - h_{w\_out}) = K_M \Delta T^n$$

Where Q is the heat transfer, $\dot{m}$ is the mass flow rate of water flowing through radiator, h_w is the water enthalpies at inlet and outlet. K_M and n are the cırve fitting coefficients obtained as a result of experiments. In order to reduce measurements uncertainities, measuremnts of each point should be carried out several times (minimum of three times). A laboratory system according to EN 442-2 is developed and a wide range of radiators are measured by using this facility. Measurement results and curve fitting coefficients are given in below table.

| | | | | Test Raport No: | 2022-227 |
| | | | | Test Raport Date: | 2.12.2022 |

Model Name	Test date	S	B	Measuremen points		
				1	2	3
Fin and tube H.E. prototype	2.12.2022					
Test Measurement info						
Air Pressure		P	kPa	101.373	101.316	101.413
Referance Air Temperature		t	°C	19.968	20.004	20.007
Water Inlet Temperature		t1	°C	86.380	74.987	52.406
Water Outlet Temperature		t2	°C	73.667	65.012	47.566
Temperature Difference		t1-t2	°C	12.713	9.975	4.840
Inlet Water Enthalpy		h1	j/kg	361.823	313.974	219.407
Outlet Water Enthalpy		h2	j/kg	308.443	272.169	199.165
Enthalpy Difference		Δh	j/kg	53.380	41.806	20.242
Mean Water Enthalpy		tm	°C	80.024	69.999	49.986
Temperature difference		Δt	°C	60.056	49.995	29.979
Water mass flow rate		qm	g/s	21.751	21.751	21.751
Thermal Output Measured		Φ	W	1161.1	909.3	440.3
Thermal Output for standard air pressure		Φme	W	1166.0	913.5	441.7

Km	n				
			Δ 60	Δ 50	Δ 30
3.689111471	1.407316089	47	1173.1	907.6	442.3

In the figures below a standard type radiator measured in the lab is shown.
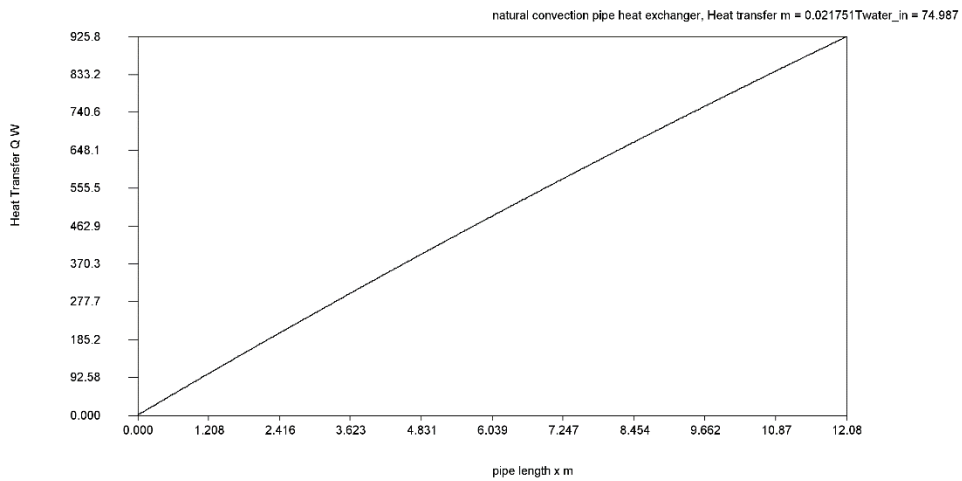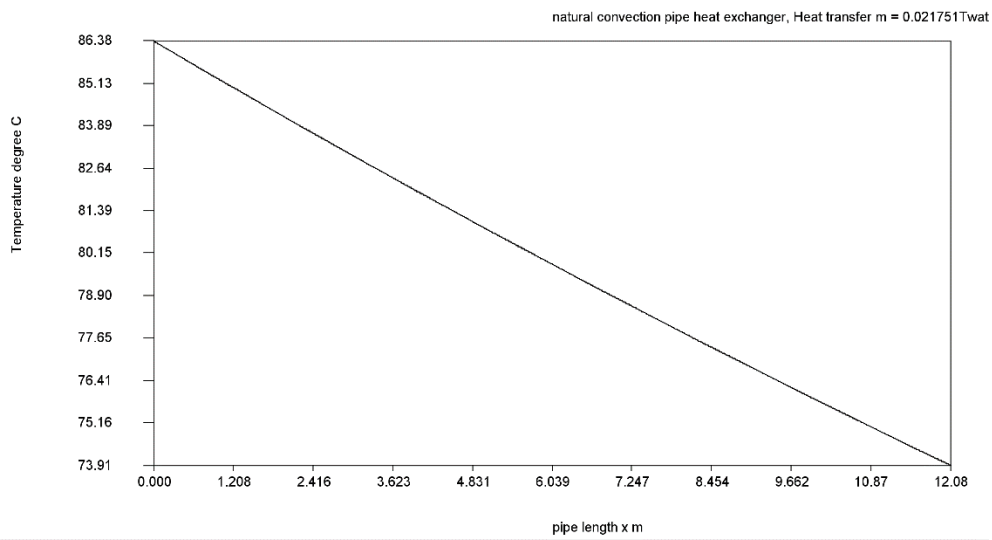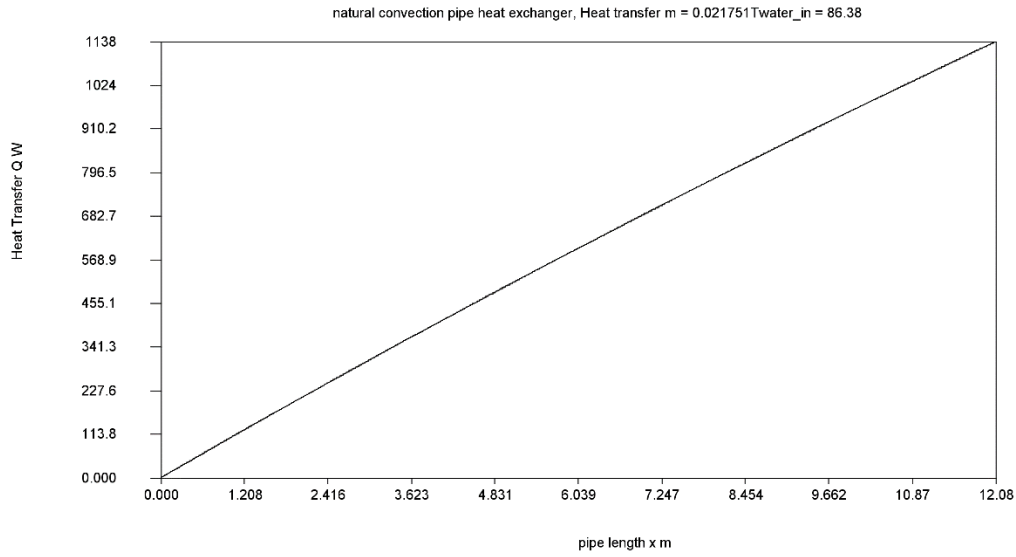

5. PROGRAM DEVELOPMENT & RESULTS

Computer codes in Java programming language is developed to calculate thermal performance of tube and fin heat exchanger. In order to calculate thermodynamic and thermophysical properties of air and water, equation of state programs are developed, and then finite difference heat transfer model of heat exchanger is developed. The computer classes used in this simulations are as follows:

Class name	
steamIAPWS_IF97	Steam-water equation of state and thermophysical properties
air_PG_CS	air equation of state (perfect gas)
HT_pipe_finned_natural	Finite difference heat transfer and heat exchanger simulation

Prototype tube-fin heat exchanger has the following properties:

Water mass flow rate	21.751×10^{-3}	kg/s
Tube Length	1	m
Tube diameter	18×10^{-3}	m
HE Height	160×10^{-3}	m
HE Width	103.93×10^{-3}	m
Fin thickness	0.79×10^{-3}	m
Distance between fins	6.85×10^{-3}	m
Pipe thickness	0.8×10^{-3}	m
Number of tubes (connected serial)	12	
Pipe material	copper	
Fin material	Aliminium	

Temperature profile for water inside tubes.

natural convection pipe heat exchanger, Heat transfer m = 0.021751Twater_in = 86.38

natural convection pipe heat exchanger, Heat transfer m = 0.021751Twat

natural convection pipe heat exchanger, Heat transfer m = 0.021751Twater_in = 74.987

natural convection pipe heat exchanger, Heat transfer m = 0.021751Twater_in = 74.987

natural convection pipe heat exchanger, Heat transfer m = 0.021751Twater_in = 52.406

natural convection pipe heat exchanger, Heat transfer m = 0.021751Twater_in = 52.406

		P	kPa	101.373	101.316	101.413
	Air Pressure	P	kPa	101.373	101.316	101.413
	Referance Air Temperature (measured and simulated)	t	°C	19.968	20.004	20.007
		t1	°C	86.380	74.987	52.406

Water Inlet Temperature (measured and simulated)					
Water Outlet Temperature (measured)	t2	°C	73.667	65.012	47.566
Water Flow (measured and simulated)	qm	g/s	21.751	21.751	21.751
Thermal Output (measured)	Φ	W	1161.1	909.3	440.3
Thermal Output (Corrected)	Φme	W	1166.0	913.5	441.7

Km	n				
			Δ 60	Δ 50	Δ 30
3.689111471	1.407316089	47	1173.1	907.6	442.3

Water outlet Temperature (simulation)	t2	°C	73.921	64.824	46.700
Thermal Output (simulation)	Φsim	W	1137.8	925.78	517.91

Water inlet Temperature (simulation)	t1	°C	90	70	50
Water outlet Temperature (simulation)	t2	°C	76.8	60.83	44.76
Room temperature (simulation)	t	°C	20	20	20
Thermal Output (simulation)	Φsim	W	1204.8	834.4	473.74

As a result, simulated temperature and heat transfer profile is closely representing the actual measured values. The model will be an imporant tool to predict performance and paremeter changes and their effects. It can be used as a first estimation toolsmfor this type of heat excahngers. Model used here has a specific geometry , bu simialar simualtion models can be developed for different geometries as well.

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